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Contagious sincerity

When should we expect partisan primaries in one party
to induce partisan primaries in the rival party?

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Abstract

Different parties can influence each other in the policy platforms they choose to compete in elections. Competition for votes creates a game-theoretic situation where each party strives to calculate the consequences of its actions while anticipating the actions of others. However, the ability and willingness to react to each other depends on the parties' sophistication, meaning whether they are sincere or strategic, which is akin to being boundedly rational or fully rational. This model studies the policies chosen by two parties in competition when one of them has a certain probability of being sincere while the other is sure to be strategic. Each party is conceived as a collection of individual members with heterogeneous preferences that choose the campaign platform in a closed primary election. Increasing the probability that one of the parties is composed of members who are sincere instead of strategic turns out to have profound consequences in the election, as this party will become more extremist. Surprisingly, this party's sincerity will have a contagious effect on the rival party, which will also become more extremist. In consequence, polarization will increase and government policy will become more volatile. These results bridge that gap in a lively empirical debate about whether primaries produce significant polarization. My theoretical results indicate that primaries may produce very large polarization, or none at all, depending on a parameter that has been previously neglected in statistical studies: the sophistication of primary voters.

Keywords: Primary elections, polarization, sincere voting, strategic voting

Resumen

Los distintos partidos políticos pueden influenciarse unos a otros al decidir sus plataformas políticas para competir en una elección. La competencia por votos crea una situación de teoría de juegos en la que cada partido busca calcular las consecuencias de sus actos así como anticipar las acciones de los demás. Sin embargo, la habilidad y la voluntad para responder unos a otros depende de la sofisticación de los partidos, es decir, si son sinceros o estratégicos, lo cual es equiparable a tener racionalidad limitada o a ser completamente racional. Este modelo estudia las políticas públicas que son escogidas por dos partidos políticos cuando uno de éstos tiene cierta probabilidad de ser sincero mientras el otro será definitivamente estratégico. Concibo a cada partido como una colección de miembros individuales con preferencias heterogéneas que escogen la plataforma de campaña en una elección primaria cerrada. Resulta que incrementar la probabilidad de que uno de los partidos esté compuesto por miembros que son sinceros en vez de estratégicos tiene consecuencias profundas en la elección, puesto que este partido se volverá más extremista. Inesperadamente, la sinceridad de este partido será contagiada al partido rival, el cual también se volverá más extremista. Como resultado, la polarización va a aumentar y las políticas gubernamentales se volverán más volátiles. Estos resultados ayudan a resolver un activo debate empírico sobre la polarización que pueden producir las primarias. Mis resultados teóricos indican que las primarias pueden producir una gran polarización, o ninguna, dependiendo de un parámetro que ha sido virtualmente ignorado en estudios estadísticos: la sofisticación de los votantes dentro del partido.

Palabras clave: Elecciones primarias, polarización, voto sincero, voto estratégico

1 Introduction: The sophistication of party members can have contagious effects

Different political parties competing in the same election can often influence each other – and they may do so in surprising ways. The academic literature has paid particular attention to one type of mutual influence, namely the parties' position-taking in the policy space. As soon as one party reveals its platform to run in the election, for example by making a public announcement or publishing a manifesto, other parties are able react by modifying their own intended platforms. The hypothesis that parties modify their policy positions in response to the policy positions of rival parties has been widely studied in academic research, both theoretically[1] and empirically.[2] This interaction between contenders is of significant consequence in elections, not least because it determines the policies that will eventually be implemented by the winner. Indeed, competition for the same prize, i.e. winning votes to get elected, provides incentives for the leaders of each party to try anticipating and outguessing the behavior of leaders of opposing parties. Coveting the same scarce resource makes elections essentially a game-theoretic situation, where each player strives to calculate the consequences of its actions relative to the actions of others.

Yet the ability and willingness of a given party to adapt to its rivals' actions can vary widely. Some parties may be run by flexible and pragmatic members who react promptly to new information, while other parties could be run by rigid or uninformed members who stick to some fundamental principles regardless of the environment. Using some common terminology in this field, we would say that different degrees of *sophistication* exist among political parties, since some of them may behave *strategically* while others may behave *sincerely*, as I will define more precisely later. I will argue that such variation can have a large impact on the way elections are conducted and resolved. In particular, we should expect the policy positions of political parties to depend on the sophistication of their members in a way that has not been fully acknowledged by the previous literature. The model in this paper finds that seemingly subtle changes in the attitude of political parties, such as choosing their platforms sincerely instead of strategically, can actually have large effects on the extremism of their policies. This matters because extremism carries a number of consequences that are ultimately suffered by common citizens, which explains why polarization in the United States and other regions has been a serious concern among scholars for many years.[3]

Primary elections are often thought to be a possible source of polarization between parties. A prevalent argument to justify this conjecture can be stated simply as follows:

[1] The vast theoretical literature studying position-taking in the policy space during electoral campaigns has its origins in Hotelling (1929) and Downs (1957).

[2] For a review of the recent empirical literature about policy shifts by parties during elections, see Adams (2012).

[3] For seminal analyses of polarization in America, see Fiorina, Abrams and Pope (2006) and McCarty, Poole and Rosenthal (2006).

primary-election voters are presumed to have more extremist preferences than general-election voters, which allegedly provides incentives for candidates to diverge away from the center in order to obtain the nomination in their respective parties. Scholars have explored this conjecture for a long time using different methodologies – but their results have remained inconclusive. In fact there is an active debate, both in the empirical and the theoretical literatures, about whether primary elections produce significant polarization or not. On the empirical side, some early contributions initially found that primary voters were indeed forcing their parties to take divergent positions;[4] but more recent analyses have instead found that primaries have no effect, or a negligible one, on polarization.[5] On the theoretical side, most of the existing formal models have predicted that primaries induce divergence of some kind;[6] but some recent theories have found general conditions under which primaries do not lead to any polarization at all.[7] A hypothesis that can reasonably be formulated, in light of these contradictory results, is that primary elections lead to polarization in some circumstances but not in others. If so, an important research question ought to be what those circumstances are. Such is the question that motivates this paper.

A general goal of this paper is to show that an understudied aspect of primary elections, namely whether the members of a party will vote sincerely or strategically, actually has a large effect on the extremism of platforms. Furthermore, I seek to prove that such extremism in one party can spread to other parties whose chosen platforms will be influenced in response. This represents a novel approach to explaining polarization in party systems such as the American one: while some aspects of this argumentation are sometimes articulated in the media by political pundits, they have not been explicitly proved in formal political theory. Some formal models have assumed sincere parties[8] while others have assumed strategic parties,[9] but only a handful has compared the two,[10] and to my knowledge they have not formally shown that increasing the expectation that one of the parties will behave sincerely

[4] Most notably Gerber and Morton (1998), Burden (2001) and Burden (2004).

[5] See for example Hirano, Snyder, Ansolabehere and Hansen (2010); Peress (2013); and McGhee, Masket, Shor, Rogers and McCarty (2014).

[6] Among many relevant models of primaries predicting some sort of divergence, see Owen and Grofman (2006); Jackson, Mathevet and Mattes (2007); Adams and Merrill (2008); Hirano, Snyder Ting (2009); Castanheira, Crutzen, Sahuguet (2010); Serra (2011); Snyder and Ting (2011); Hummel (2013); Adams and Merrill (2014); Hortala-Vallve and Mueller (2015); Kselman (2015a); Kselman (2015b); Ting, Snyder and Hirano (2015); Amorós, Puy and Martínez (2016); Grofman, Troumpounis and Xefteris (2016); Woon (2016).

[7] The few models finding complete convergence in spite of primaries include Kselman (2015b); Serra (2015); Woon (2016); and Serra (2017).

[8] Models assuming sincere primary voters include Adams and Merrill (2008) and Grofman, Troumpounis and Xefteris (2016).

[9] Models assuming strategic primary voters include Owen and Grofman (2006); Jackson, Mathevet and Mattes (2007); Hirano, Snyder Ting (2009); Hummel (2013); Hortala-Vallve and Mueller (2015); Serra (2015); Ting, Snyder and Hirano (2015); Amorós, Puy and Martínez (2016); Serra (2017).

[10] The few previous models contrasting the sincere and strategic motivations of primary voters include Adams and Merrill (2014); Kselman (2015a); Kselman (2015b) and Woon (2016). These models address a diversity of questions that are fairly different from the main questions in this paper.

instead of strategically will significantly increase polarization.[11] My model is also original in analyzing the contagious effect that one party's sophistication will have on the other party's extremism. Finally, a feature of this model is to analyze the behavior of individual party members with different ideal points, which adds a layer of mathematical complexity above studying parties as large groups of homogenous voters.[12] These theoretical features have empirical implications of relevance, as I will discuss in the concluding section of the paper.

To study the effect that sincere or strategic members in one party can have on all parties, I develop a model in this paper with the following features. I will consider a two-party system but I do not focus on parties per se as unitary actors; rather, I focus on the actions of their individual members in a primary election. Each member of a party has an ideal point in the left-right policy space: the median member of one party has an extreme-left ideal point while the median member of the other party has an extreme-right ideal point. Each party chooses its campaign platform democratically in a closed primary election among its members. I consider two types of voters in primary elections: strategic ones and sincere ones. By "strategic" I mean that a primary voter takes into account the expected platform of the rival party in forming her preferences for her own party's platform. In contrast, by "sincere" I mean that a primary voter's preferences for her party's platform are always identical to her original preferences regardless of the expected platform of the rival party. A useful way to distinguish both attitudes is to equate strategic voting with full rationality and sincere voting with bounded rationality. Indeed, a member of a political party is said to vote strategically if she incorporates in her calculations all the information available, including the game-theoretic interaction with the members of rival parties. On the other hand, she is said to vote sincerely if she fails to make such calculations. The empirical literature on political behavior has consistently documented the existence of both types of voting among party members,[13] so it is worth incorporating them in our formal theories. Unfortunately, the empirical literature has paid almost no attention to why and when strategic voting will be high or low compared to sincere voting.[14] Whatever the reasons, I will assume that one of

[11] Among those papers comparing sincere and strategic voting in primaries, Woon (2016) is closest in spirit to mine. In a model with a right-wing party and a left-wing party with homogenous members, he finds that fully strategic parties will completely converge to the center (which is akin to the result in Serra (2015)). However, he finds that introducing some amount of sincerity as "noise" in the party members preferences will produce a more polarized election. Importantly, he tests this hypothesis with an experiment showing that primary elections in the laboratory do indeed create some divergence, perhaps due to the sincerity of the subjects.

[12] Other models of primaries analyzing heterogeneous party members are Owen and Grofman (2006); Jackson, Mathevet and Mattes (2007); Adams and Merrill (2008); Hummel (2013); Adams and Merrill (2014); Kselman (2015a); Kselman (2015b); Amorós, Puy and Martínez (2016); Grofman, Troumpounis and Xefteris (2016).

[13] A classic reference is Abramson, Aldrich, Paolino and Rohde (1992). For more recent research, see Hall and Snyder (2015) and the citations therein.

[14] A notable exception is Hall and Snyder (2015) who study whether different contexts lead to more sincere voting in primaries. These authors find that in elections where the information to voters is low due to limited media coverage, such as in local elections instead of federal ones, there are more wasted ballots due to sincere voting.

the parties has a known probability of choosing its platform sincerely instead of strategically, while the other party always chooses its platform strategically. Finally, I assume that parties hold their primaries in sequence: a fully strategic party decides first, while a party that might be fully strategic or fully sincere decides subsequently. Hence the second party can react to the previous decision taken by the first one, but it will do so only if it is strategic rather than sincere. This uncertainty about the sophistication of the second party will induce the first party to hedge its platform in a forward-looking optimization that will have surprising implications.

The results will shed light on the complex relationship between primary elections and polarization. During primaries, I prove that the preferred platform of the median party member is a Condorcet winner in both types of parties: sincere ones and strategic ones. In contradiction to much of the existing literature, I find no polarization whatsoever if the primary voters in both parties are known to be fully strategic.[15] But polarization arises as soon as one party is expected to be sincere with some probability. In fact, the likelihood that primary voters in one party might behave sincerely drives both parties to choose extremist platforms. If this likelihood is large enough, I find the surprising result that both parties will behave as if they were fully sincere even if one of them is not; which I will call "contagion" throughout the paper. As a consequence polarization will be extreme. So the mere expectation (even if it is not realized) that one party might be sincere instead of strategic, is enough to bring polarization from its minimal value of zero to extreme values. A general conclusion of this research is that seemingly innocuous changes in the attitude of primary voters can have large effects on policy platforms.

The subsequent sections of the paper proceed as follows. I introduce the main assumptions and definitions of the model, including my treatment of sincere and strategic behavior. I analyze the primary elections in the second-moving party in both scenarios where its members behave sincerely or strategically. Then I analyze the primary election in the first-moving party which is known to be strategic. Finally, I derive the equilibrium results in this election as a function of the main exogenous variable: the probability that the second-moving party will be sincere. A concluding section discusses the relevance of the results for the theoretical and empirical literatures on primaries. An appendix posted online contains the proofs of all the results.

[15] This is consistent with the recent results in Kselman (2015b), Serra (2015), Woon (2016) and Serra (2017), which find in somewhat different models that the rationality of political parties should drive them to the center of the spectrum, even if they hold primaries.

2 Assumptions, definitions and timing

2.1 General electorate

This election takes place in a unidimensional policy space interpreted as the left-right political spectrum. A specific policy located in this spectrum is labeled x, with $x \in \mathbb{R}$. There is an electorate that will vote to choose the party that will take power. All citizens have single-peaked and quadratic utility functions with ideal points located on this dimension. The electorate has a median voter who will be decisive in the election: whatever party she votes for will win the election. We will call M this median voter and I will assume that her ideal point is publicly known with certainty to everyone – so this model does not display uncertainty about the preferences of voters. We will call $U_M(x)$ the utility function of M and her ideal point will be adjusted to zero. In sum, we have:

$$U_M(x) = -x^2$$

2.2 Parties

Two parties compete in this election: a right-wing party labeled R, and a left-wing party labeled L. In this paper, each party is conceived as a collection of members who decide the party platform democratically in a closed primary election.[16] Party R consists of n_R members while party L consists of n_L members, where each of these numbers can be arbitrarily large; in fact they could be in the millions for national parties. I will only assume that n_R and n_L are odd. The members of each party can be ordered from most extremist to most centrist, according to their ideal points. We will call r_i the ideal point of the i^{th} member of party R, and we will call l_j the ideal point of the j^{th} member of party L. In a slight abuse of language, I will use the same notation for the members holding such ideal points. Hence r_i and l_j will both refer to the i^{th} member of party R and the j^{th} member of L, as well as to their ideal points. They all have single-peaked and quadratic utility functions of the following kind:

$$\begin{aligned} U_{r_i}(x) &= -(r_i - x)^2 \\ U_{l_j}(x) &= -(l_j - x)^2 \end{aligned}$$

For simplicity, I will assume that party R does not have left-wing members and party L does not have right-wing members, meaning that $r_i \geq 0$ and $l_j \leq 0$ for every i and j. The distribution of ideal points in each party has a unique median; we will call r_M the median member of party R, and l_M the median member of party L. I assume that the exact values of

[16] Given the specific goals of this paper, I am therefore ignoring the important role that party bosses, donors and other elites have been proved to play in primaries. For theories incorporating those elites, see Crutzen, Castanheira and Sahuguet (2010); Serra (2011); and Hortala-Vallve and Mueller (2015).

r_M and l_M are publicly known with certainty to everyone. To simplify the analysis, instead of working with abstract values for r_M and l_M, I will adjust the location of the median party members of R and L to one and minus one, respectively, meaning that $r_M = 1$ and $l_M = -1$. For interpretation purposes, we should think of the locations -1 and 1 as being quite extreme on the left and the right of the political spectrum.[17] Hence the party medians have the following utility functions:

$$\begin{aligned} U_{r_M}(x) &= -(1-x)^2 \\ U_{l_M}(x) &= -(-1-x)^2 \end{aligned}$$

To summarize, each party can be described as a collection of members with the following characteristics:

$$\begin{aligned} R &= \{r_1, r_2, ..., r_M, ..., r_{n_R}\} \text{ with } 0 \leq r_1 \leq r_2 \leq ... \leq r_M = 1 \leq ... \leq r_{n_R}. \\ L &= \{l_1, l_2, ..., l_M, ..., l_{n_L}\} \text{ with } 0 \geq l_1 \geq l_2 \geq ... \geq l_M = -1 \geq ... \geq l_{n_L}. \end{aligned}$$

2.3 Primary elections

Both parties must design their platforms to compete in the election. These platforms consist on locations in the left-right political spectrum that are offered to the electorate. We will call x_R the platform adopted by party R, and x_L the platform adopted by party L. As is standard in the literature, I will assume that each platform becomes binding once it is announced, representing the policy that will effectively be implemented if the respective party wins the election.

Each party's platform is decided democratically in a closed primary election among all its members. I will assume that whenever a Condorcet winner exists among all the policies in the political spectrum, then it will become the party's platform. In other words, suppose that each party member has made up her mind about her preferences for her party's platform. If there exists a policy x that is strictly preferred by a strict majority of party members to every other policy x', then x will become the party's platform. One way to justify this decision process is interpreting it as the result of an internal contest among ambitious candidates seeking the nomination, which I do not explicitly model here.[18] Then this assumption

[17] This adjustment to 1 and -1 represents a slight loss of generality because it imposes that both parties have median members exactly at the same distance from the center. It can be readily proved that all the results would go through with more abstract ideal points, for example with arbitrary $r_M > 0$ and $l_M < 0$. However, making this adjustment is useful as it will shorten the presentation and make the results more intuitive. Another advantage of adjusting the median members of each party to symmetric values, is that any asymmetries that we find from the model can be attributed to other parameters of interest, not to the ideal points.

[18] For papers that do model the interaction between candidates strategically choosing where to locate ideologically during primaries, see Adams and Merrill (2008); Hummel (2013); Adams and Merrill (2014); Kselman (2015a); Kselman (2015b); Serra (2015); and Grofman, Troumpounis and Xefteris (2016).

can be interpreted as the result of competition among several hopefuls who are seeking the nomination: through an exploration process, I assume that one of them will find the Condorcet winner and will be nominated with such platform.

For presentation purposes, I will impose the following restrictions on the platforms that can be announced by parties: $x_R \in [0, 1]$ and $x_L \in [-1, 0]$. So the right-wing party must announce a right-wing platform that is no more extreme than the ideal point of its median member, and similarly for the left-wing party. These conditions do not need to be assumed, as they can be proved to be true in equilibrium. In fact, I do not use them in the derivations of the main outcomes. However, assuming them at the outset allows the presentation to be cleaner and it streamlines the results.

2.4 Sophistication of party members

A main goal of this paper is to compare strategic voting and sincere voting in primary elections. While these concepts may seem fairly intuitive, in fact different definitions can be found in the theoretical literature, and different types of operationalization can be found in the empirical literature.[19] So it is worth being precise in the use of our terms, as I attempt to do in these definitions:

Strategic and sincere behavior: *In this context, we say that a party member is* strategic *if her preferences regarding her party's platform correspond to her expected payoff from each possible platform, calculated using all available information such as the probability of winning the election given the rival party's expected platform. In contrast, we say that a party member is* sincere *if her preferences regarding her party's platform correspond to her original utility function over policy, ignoring all other information such as the probabilities that different platforms have of winning the election given the rival party's expected platform.*

This paper seeks to identify the impact of one party choosing its platform sincerely instead of strategically, while keeping the distribution of its members' ideal points constant. In particular, I would like to study the effect of this change of attitude on this party's platform; on the other party's platform; on the polarization between the two parties; and on the definitive platform implemented after the election. For this purpose, I will assume that one of the parties, namely the left-wing party L, has a certain probability of being sincere and a certain probability of being strategic. We will call σ this probability which is interpreted as a "prior belief" by all players – hence there is a prior belief σ that party L has sincere members, and a prior belief $1 - \sigma$ that is has strategic members, with $\sigma \in [0, 1]$. In contrast, the right-wing party R is known to have strategic members for sure. I should

[19] For a discussion, see Hall and Snyder (2015).

note that in this model either all the members of the same party are strategic or they are all sincere. Thus, to simplify the analysis, I am avoiding the situation where the same party has a mix of sincere and strategic members.[20]

To be more concrete about the meaning of strategic and sincere behavior, let me state the specific maximization problems that different party members have in mind. The crucial difference is whether they are willing and able to take the final outcome of the election in consideration for their preferences. We will call x^* the winning platform in the election; this is the policy that will be implemented by the winning party. There are two possibilities for a member l_j of party L: either she will be sincere or she will be strategic. If the members of party L turn out to be sincere, they will fail to incorporate the consequences of choosing a certain platform x_L for the outcome of the election, and instead they will behave as if they were maximizing the following utility function that myopically ignores x^* :

$$\max U_{l_j}\left(x_L\right) = -\left(l_j - x_L\right)^2 \tag{1}$$

In contrast, if the member l_j of party L is strategic, her preferences will depend on the final outcome of the election, x^*, so she wishes to maximize the following utility function:

$$\max U_{l_j}\left(x^*\right) = -\left(l_j - x^*\right)^2 \tag{2}$$

Likewise, all the members of party R are assumed to be strategic, so each member r_i wishes to maximize the following payoff which depends on x^*:

$$\max U_{r_i}\left(x^*\right) = -\left(r_i - x^*\right)^2 \tag{3}$$

It should be noted, however, that members of R will actually face a decision under uncertainty. So, as we will see later on, their maximization objective is actually the expectation of the utility U_{r_i} mentioned above.

2.5 Timing and solution concept

The sequence in this game consists on the following stages:

1. The members of party R, which are strategic, choose their party's platform x_R by majority voting at a closed primary election.

2. Nature chooses whether all the members of party L are sincere with probability σ, or strategic with probability $1 - \sigma$.

[20] Such situation is mathematically complex, and leads to analyzing within-party effects whereas my focus in this paper is between-party effects, such as the contagion of one party's sincerity on the other one. A companion paper to this one is devoted to the full-fledged analysis of a party with a mixture of sincere and strategic members.

3. The members of party L choose their party's platform x_L by majority voting at a closed primary election.

4. The general electorate votes for R or L, and the winning party implements its promised campaign platform.

Since this is a sequential game, it must be solved by backward induction. The solution concept is subgame-perfect equilibrium and we assume there is no abstention and voters never use a weakly dominated strategy. Accordingly, we must start by solving the game at the last stage, taking the results as given when we analyze the reduced game at the earlier stage, and so on.

3 The general election

The fourth stage is generally easy to predict as the median voter will always choose the party whose platform is closest to her ideal point. But a complication arises when both parties are exactly equidistant, making the median voter indifferent, which will actually occur in several equilibria. In this case we need to make indifference assumptions to solve the game. A frequent assumption in models of this kind is that both parties will win with equal probability – but in this model, such assumption is not so realistic, in addition to preventing any equilibria to exist. Instead, based on the likely behavior of all players, I will make the following two indifference assumptions, labeled IA1 and IA2.

Indifference Assumptions: *If the median voter M is indifferent between L and R, she will:*

(IA1) *Vote for R if L is a sincere party*

(IA2) *Vote for L if L is a strategic party*

Both assumptions could be justified with the following considerations. Suppose that M is expected to be indifferent because both parties are planning to adopt equidistant platforms from the center; in other words we are expecting $x_L = -x_R$. The first indifference assumption, IA1, is reasonable because R is strategic while L is sincere. In this case, L will rigidly adopt its chosen platform regardless of any other calculations. As we will see in the analysis below, given that R is forward looking, it can calculate the platform that L will rigidly adopt. We can assume that R will actually choose a platform $x_R = -x_L - \varepsilon$ that is infinitesimally more centrist than x_L to break the tie in its favor. The second indifference assumption, IA2, is reasonable because L moves last. If L is strategic we can assume that it will actually choose a platform $x_L = -x_R + \varepsilon$ that is infinitesimally more centrist than the platform x_R that R chose previously. This will break the tie in favor of L. Finally, I should note that making other assumptions, such as a coin toss in case of indifference, might prevent any equilibria from existing, but the behavior of all players would still converge ever so closely to the equilibria that I find here with IA1 and IA2.

The third stage must be solved in two scenarios, depending on Nature's decision at the second stage of whether all the members of party L will be sincere or strategic. In either case, we must predict the result of L's primary election. We will call x_L^* the equilibrium platform chosen by the members of L. At the first stage there is a primary election in party R, but given that its members are strategic they will calculate the likely consequences of their decision at subsequent stages. We will call x_R^* the equilibrium platform chosen by the members of R. Finding x_L^* and x_R^* will allow us to compute the policy implemented after the election, x^*. However, all these results will depend on the context. The main exogenous parameter in this model is σ, which is interpreted as the perceived likelihood *ex-ante* that party L will choose its platform sincerely instead of strategically. The goal is to make comparative statics on this variable.

4 L's primary election with sincere party members

There is a chance that party members in L will behave sincerely in their primary election at the third stage of the game; this will be decided by Nature with probability σ at the second stage. In this case, the members of L will act only on their original ranking of platforms, regardless of the consequences of their choice. In particular, they will ignore whether their preferred platform would lose the election for sure because the rival party R has adopted a more centrist one. To be concrete, each member l_j will evaluate a platform x_L solely according to the formula given in equation 1. These preferences are single-peaked and symmetric around the ideal point of each party member. Therefore, in comparing two different alternatives, each party member will always rank highest the one that is closest to her ideal point.

Among all party members, we will first focus on the median l_M. It will be particularly relevant to keep in mind her preferred platform, which is stated in lemma 1. This will serve later on as a point of comparison to her preferred platform if she was strategic instead of sincere.

Lemma 1 *The following will hold irrespective of the platform x_R that party R adopted previously. If L's median member, l_M, is sincere, then the platform that she would prefer her party to adopt is her ideal point: $x_L = -1$.*

Now we must study how the behavior of all other members relates to their median. Recall that L's platform will be decided democratically in a closed primary election. I have assumed that an alternative that could beat every other alternative in a pairwise vote, if it exists, will become the winner at the primary contest. Theorem 1 states that such a platform exists; not surprisingly, it will be the alternative preferred by the median party member. This result is simply an application of the standard median voter theorem as originally stated by Duncan Black (1958).

Theorem 1 *The following will hold irrespective of the platform x_R that party R adopted previously. If all the members of party L behave sincerely in their primary election, then the ideal point of L's median member, $l_M = -1$, is a Condorcet winner and will become the party's platform to compete in the general election. Therefore $x_L^* = -1$.*

With these results, we can calculate the outcome of the election and the ensuing payoffs to all players. We just proved that a party L with sincere members will end up adopting the median member's ideal point as its platform; namely $x_L = -1$, regardless of R's platform. Recalling our assumption that $x_R \in [0, 1]$, we can deduce the winner of this election: it will always be party R. The reason is that the members of party R, who are strategic, will choose a more centrist platform than the members of party L, who are sincere.[21] This is summarized in corollary 1.

Corollary 1 *The following will hold irrespective of the platform x_R that party R adopted previously. If all the members of party L behave sincerely in their primary election, then party R will win the general election, and the platform implemented will be $x^* = x_R$. The payoffs to the members of each party will be*

$$\begin{aligned} U_{r_i}\left(x^* : x_R,\ L \text{ is sincere}\right) &= U_{r_i}\left(x_R\right) = -\left(r_i - x_R\right)^2 \\ U_{l_j}\left(x^* : x_R,\ L \text{ is sincere}\right) &= U_{l_j}\left(x_R\right) = -\left(l_j - x_R\right)^2 \end{aligned}$$

Note that in this corollary I have stated the actual payoffs that members of party L would receive after the election, meaning $U_{l_j}\left(x^*\right)$ as in equation 2, while in practice these members are behaving as if they wanted to maximize $U_{l_j}\left(x_L\right)$ as in equation 1. This contrast between the goal pursued by a member l_j and the goal she should pursue underlines that sincere voting can be interpreted as a manifestation of bounded rationality.

5 L's primary election with strategic party members

There is also chance that party members in L will behave strategically in their primary election at the third stage of the game; this will be decided by Nature with probability $1 - \sigma$ at the second stage. In this case, the members of L will take into account the consequences of their party's choice for the election result. In particular, they will calculate whether their

[21] In case party R adopted exactly the platform $x_R = 1$, parties would be symmetric around the median voter in the general election, but R would still win the election given our indifference assumption IA1.

preferred alternatives would lose the election for sure because the rival party R adopted a more centrist one. To be concrete, each member l_j is now driven by the objective function in equation 2, which depends on the winning platform x^*.

So in this section it is actually necessary to determine what the winning platform will be. Recall that a party is sure to win the election if it is located strictly closer to zero than its rival. If both platforms are equidistant from zero, then we need to appeal to the indifference assumptions that I stated above. Given that L is strategic, the assumption IA2 applies in this case; therefore a tie would break in favor of party L who would win the election. This is summarized in the following values for the winning platform, x^*:

$$x^* = \begin{cases} x_R \text{ if } 0 \leq x_R < -x_L \\ x_L \text{ if } 0 \leq -x_L \leq x_R \end{cases}$$

Members of party L would like to find the platform x_L that would ensure the most favorable outcome x^* for them, given the observed platform x_R that was previously chosen by R. In other words, for each party member the optimal choice x_L^* needs to be a *best response* to the observed platform x_R. Again, among all party members, we will first focus on the median l_M whose preferred platform is not so trivial in this case. We should first note that l_M's ideal point is not necessarily her preferred choice. In fact, she would *almost never* want her party to adopt her ideal point as its platform because this would lead to defeat in the general election. As can be seen from the value of x^* above, if party L wishes to win the election in order to prevent party R from implementing its policies, it must choose a platform that is not more extremist than x_R. The solution to this optimization problem for the median member is given in Lemma 2.

Lemma 2 *Assume that party R previously adopted a specific platform x_R, with $x_R \in [0, 1]$. If L's median member, l_M, is strategic, then the platform that she would prefer her party to adopt is $x_L = -x_R$.*

Note that l_M's preferred platform, now that she is strategic, is in general more centrist than it was in the previous section when she was sincere. This is true in spite of her ideal point being the same. According to lemma 1, the median member would want her party to adopt the platform $x_L = -1$ if she was sincere; but according to lemma 2, she would want her party to adopt a platform $x_L = -x_R \geq -1$ if she was strategic.

The median party member of L will play again a crucial role in the primary election. Theorem 2 states that her preferred platform will be a Condorcet winner – however, her preferred platform is not her ideal point anymore, but the result of a careful optimization problem that best responds to the environment.

Theorem 2 *Assume that party R has adopted a specific platform x_R, with $x_R \in [0, 1]$. If all the members of party L behave strategically in their primary election, then the optimal* *platform for L's median member, $-x_R$, is a Condorcet winner and will become the party's platform to compete in the general election. Therefore $x_L^* = -x_R$.*

Compared to the results in the previous section, we can immediately remark that party L would in general adopt a more centrist platform in this section (comparing theorem 1 to theorem 2). This holds in spite of the ideal points being distributed the same way in both cases. We can now use these results to calculate the outcome of the election and ensuing payoffs to all players. We just proved that a party L with strategic members will end up adopting the median member's optimal platform given the platform x_R that party R had adopted previously, which is $x_L = -x_R$. Recalling our indifference assumption IA2 we can deduce the winner of this election: it will be party L. These remarks are formalized in corollary 2.

Corollary 2 *Assume that party R has adopted a specific platform x_R, with $x_R \in [0,1]$, at a previous stage. If all the members of party L behave strategically in their primary election, then party L will win the general election, and the platform implemented will be $x^* = -x_R$. The payoffs to the members of each party will be*

$$\begin{aligned} U_{r_i}\left(x^* : x_R,\ L \text{ is strategic}\right) &= U_{r_i}\left(-x_R\right) = -\left(r_i + x_R\right)^2 \\ U_{l_j}\left(x^* : x_R,\ L \text{ is strategic}\right) &= U_{l_j}\left(-x_R\right) = -\left(l_j + x_R\right)^2 \end{aligned}$$

6 R's primary election

We can finally start solving the game at its first stage, i.e. the primary election inside party R. We are assuming that all the members of party R are strategic in the sense of following the objective function in equation 3. We will also assume that all the members of R are forward looking since they take into account the consequences of their choices in subsequent stages of the game. In particular, each member of R will evaluate any possible platform x_R according to its probable impact on party L's platform, x_L, knowing that both platforms together will determine the winning platform x^*.

One of the complications of R's decision is that L could be sincere or strategic, as this can completely change the final outcome of the election. Given a choice of platform x_R by party R, we know from our previous analysis that party L will in general choose quite a different platform x_L in either case. According to corollary 1, a sincere L will adopt the ideal point of its median member which is an extreme-left position, thus allowing R to win. But according to corollary 2, a strategic L will optimally choose the exact opposite of R's platform to win the election most favorably. This is summarized in the following values for the winning policy x^* as a function of R's platform and L's sophistication:

$$x^* = \begin{cases} x_R \text{ if } L \text{ is sincere} \\ -x_R \text{ if } L \text{ is strategic} \end{cases}$$

The primitive variable in this model is σ, the prior belief that L will be sincere. So we would like to find the optimal choice by R for each possible value of σ, with $\sigma \in [0,1]$. In effect, the members of party R are facing a decision under uncertainty whereby they need to maximize their payoffs in expected form. To understand their actual goal, we need to specify the expected utility to each member r_i from each platform x_R. This expected utility will be called $EU_{r_i}(x^* : x_R)$, and will be given by the following formula:

$$\begin{aligned} EU_{r_i}(x^* : x_R) &\equiv \sigma U_{r_i}(x^* : x_R,\ L \text{ is sincere}) + (1-\sigma) U_{r_i}(x^* : x_R,\ L \text{ is strategic}) \\ &= -\sigma (r_i - x_R)^2 - (1-\sigma)(r_i + x_R)^2 \end{aligned} \tag{4}$$

Among all party members, we will first focus on solving this optimization problem for the median r_M. Lemma 3 provides the platform that r_M would find optimal given the uncertainty about L's sophistication.

Lemma 3 *Recall that r_M, the median member of party R, is assumed to be strategic. The platform that r_M would prefer her party to adopt is* $x_R = \begin{cases} 0 \text{ if } \sigma \in \left[0, \frac{1}{2}\right] \\ 2\sigma - 1 \text{ if } \sigma \in \left(\frac{1}{2}, 1\right] \end{cases}$

Next section of the paper has a longer discussion to interpret and explain the main results of the model, but some remarks can briefly be made here about this lemma. In particular, we should note the difference in the preferred platform x_R for low and high values of σ. When there is a relatively low probability that party L will be sincere, namely $\sigma \in \left[0, \frac{1}{2}\right]$, the platform preferred by the median member of party R is a corner solution consisting on locating exactly at the center of the political spectrum. In contrast, when there is a relatively high probability that party L will be sincere, namely $\sigma \in \left(\frac{1}{2}, 1\right]$, the platform preferred by the median member of party R is an interior solution consisting on locating at a more partisan platform, and increasingly so with σ.

This result will play a crucial role in solving the game, as the preferences of r_M will turn out to be pivotal in R's primary election. Remember that, among all possible policies, party R will choose its platform democratically by espousing the Condorcet winner, if there is any. Theorem 3 states that the median member's preferred platform will win the primary – however, her preferred platform is not her ideal point, but the result of an optimization under uncertainty that carefully balances out the probable outcomes in subsequent stages.

Theorem 3 *Recall that all members of party R are assumed to behave strategically in their primary election. The optimal platform for R's median member, as given in lemma 3, is a*

Condorcet winner and will become the party's platform to compete in the general election. Therefore

$$x_R^* = \begin{cases} 0 \text{ if } \sigma \in \left[0, \frac{1}{2}\right] \\ 2\sigma - 1 \text{ if } \sigma \in \left(\frac{1}{2}, 1\right] \end{cases}$$

This finalizes the analysis of all the players' behavior in this game, which allows deriving all the outcomes of the election as a function of the exogenous variables. I devote next section to presenting, discussing and interpreting the main outcomes in the model.

7 Extremism and polarization due to primary elections

7.1 Equilibrium outcomes

The previous results determine the behavior of all players throughout the game. Importantly for the purpose of this paper, this behavior can be stated as a function of the main exogenous parameter in this model: the prior probability σ that the left-wing party L will be sincere rather than strategic. This will allow us to make comparative statics on the main variables, namely the parties' equilibrium platforms, x_R^* and x_L^*, and the winning policy in equilibrium, x^*. Theorem 4 starts by providing the outcomes of this election relative to Nature's decision at the second stage of the game.

Theorem 4 *For each value of σ there exists a unique set of equilibrium strategies in this game. The equilibrium strategies and outcomes are given in table 1.*

Table 1: Equilibria in this election as a function of σ

	If L is sincere	If L is strategic
$\sigma \in \left[0, \frac{1}{2}\right]$	$x_R^* = 0$ $x_L^* = -1$ Elected party: R $x^* = 0$ (see Figure 1)	$x_R^* = 0$ $x_L^* = 0$ Elected party: L $x^* = 0$ (see Figure 2)
$\sigma \in \left(\frac{1}{2}, 1\right]$	$x_R^* = 2\sigma - 1$ $x_L^* = -1$ Elected party: R $x^* = 2\sigma - 1$ (see Figure 3)	$x_R^* = 2\sigma - 1$ $x_L^* = -2\sigma + 1$ Elected party: L $x^* = -2\sigma + 1$ (see Figure 4)

The four scenarios described in table 1 can be visualized in the following graphs. Figures 1-4 depict the equilibrium policies that we can expect depending on the value of σ and the realization of L's sophistication.

Figure 1

$\sigma \in \left[0, \frac{1}{2}\right]$ **and** L **is sincere**

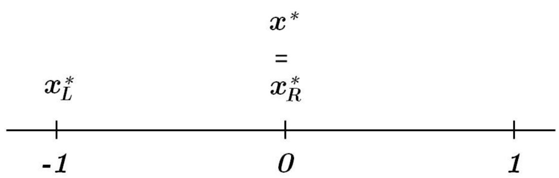


Figure 2

$\sigma \in \left[0, \frac{1}{2}\right]$ **and** L **is strategic**

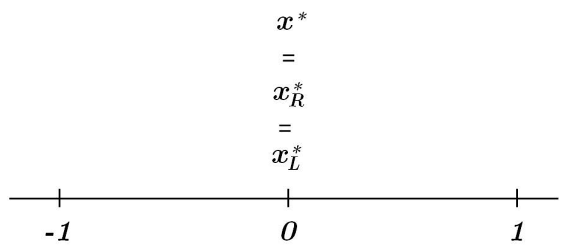


Figure 3

$\sigma \in \left(\frac{1}{2}, 1\right]$ **and** L **is sincere**

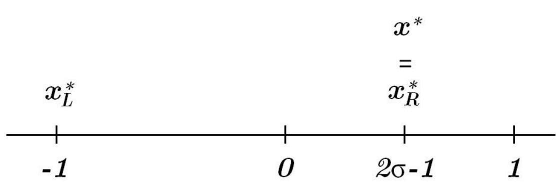
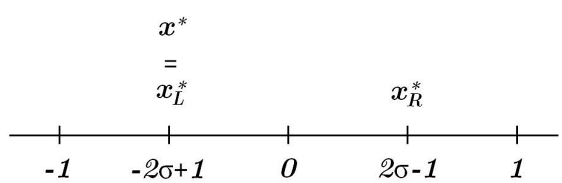


Figure 4

$\sigma \in \left[0, \frac{1}{2}\right]$ **and** L **is strategic**



7.2 Extremism of policy positions

A main interest of this paper is to analyze formally some circumstances that may actually lead primary elections to induce extremism. Specific attention is paid to the effect that voters' sophistication may have on their party's platform, and perhaps on the rival party's platform via contagion. Theorem 4 establishes that such effects do occur and, at least theoretically, they can be very large. Here are some interpretations of these results.

If the probability that L will be sincere is sufficiently small (namely $\sigma \leq \frac{1}{2}$), then R will converge completely to the median voter. This happens for two reasons. The first reason is that choosing $x_R^* = 0$ eliminates any uncertainty since the result would be $x^* = 0$ for any realization of L's sophistication. In contrast, choosing any platform with strictly positive value induces a lottery between x_R^* with probability σ, and $-x_R^*$ with probability $1 - \sigma$. The reason is that with probability σ party L will be sincere and will lose the election, and with probability $1 - \sigma$ party L will be strategic and will win the election. This lottery is not worth it for $\sigma < \frac{1}{2}$ because the probability of beating L is too low, which yields a lower expected *value* than the risk-free option. There is a second reason that reinforces the first one, which is the risk aversion of party members since I have assumed their utility functions to be quadratic. Given this concavity, the expected *utility* from the lottery is even lower then the expected *value*. So the members of R prefer to take no risk by choosing the sure value of zero. At the subsequent stage, with a high probability, L will be strategic and hence will also converge completely to the median voter such that the general electorate will face two identical centrist parties. With a small probability, L will be sincere and will diverge completely to its ideal point such that the general electorate will face a completely centrist party and a completely leftist party.

If the probability that L will be sincere is sufficiently large (namely $\sigma > \frac{1}{2}$), then R will be sensitive to σ. In fact $x_R^* = 2\sigma - 1$, which is an increasing function. This means that R will be increasingly extremist as its expectation that L is sincere increases. We should note that this occurs independently of the the distribution of ideal points r_i. To be clear, we are continually assuming that the members of party R are fully strategic, and we are keeping their ideal points constant; and yet, they are choosing an increasingly extremist platform as it becomes increasingly likely that the rival party L will be sincere. This is exactly what I have dubbed *contagion* in this paper. Such behavior from party R comes from its decision under uncertainty which must balance out winning with a favorable policy x_R^* against losing to an unfavorable policy $-x_R^*$. Its probability of winning is σ, so as this probability increases, it becomes more attractive to adopt a radical-right platform even if this entails the risk of losing against a radical-left platform. However, the benefit of a more favorable lottery is offset by risk aversion. Given the concavity of its utility function, party R would in general not increase its platform all the way to its ideal point, because eventually the disutility from $-x_R^*$ becomes too large compared to the utility of x_R^*, even knowing that the former has a lower probability than the latter. This allows for an interior solution. At the subsequent stage, with a high probability, L will be sincere and will diverge to its ideal point such that the general electorate will face an extreme left-wing party and a center-right party. With a low probability, L will be strategic and will diverge only up to $-x_R^*$, such that the general electorate will face a center-left party and a center-right party.

These dynamics lead to a broad range of predictions in terms of parties' locations. By calibrating the primitive variable σ, this model can explain a wider range of outcomes than models of spatial elections usually can. The type of equilibria that can arise in this model include the following:

- A completely centrist party facing an extremist party (figure 1)
- Two completely centrist parties (figure 2)

- A moderately partisan party facing an extremist party (figure 3)
- Two moderately partisan parties (figure 4)
- Two extremist parties (a limit version of figure 3)

7.3 Polarization between parties' platforms

A related interest in this paper is whether primaries will create polarization between the parties' platforms, and if so, whether it will be significant or negligible. In all the equilibria reported in theorem 4, we can compare the campaign platform of party R with that of party L to determine whether they are located far apart. For precision, I will call polarization π, and I will define it as the distance between the equilibrium platforms of both parties, meaning that $\pi \equiv |x_R^* - x_L^*|$. If polarization is defined this way we can begin talking about levels of polarization that are "small" or "large". The smallest value that π can take is zero, which corresponds to both parties converging completely to each other by adopting the same platform. I will call $\pi = 0$ the "minimal polarization." On the other hand, the value of π could in principle be arbitrarily large, even converge to infinity. But actually polarization in this model should not realistically be expected to be larger than two: this corresponds to both parties diverging from the center all the way to the ideal points of their median members, which is as far as a primary election would realistically push parties. I will call $\pi = 2$ the "maximal polarization."

The equilibria in this game, however, present uncertainty about the final outcomes. Even knowing the platform x_R^* that will be chosen by party R, there is still ambiguity about how extreme the platform x_L^* will be depending on whether Nature chooses party L to be sincere or strategic. Given the different probabilities of these two scenarios, the game presents some uncertainty *ex-ante* about how polarized the platforms will be *ex-post*. To address this ambiguity, I will define *average polarization* as the distance that should be expected between the platforms of both parties before the actual sophistication of party L has been revealed. I will label average polarization $E\pi$, and will define it exactly as follows:

$$\begin{aligned} E\pi &\equiv \sigma \left(\pi : L \text{ is sincere}\right) + (1-\sigma)\left(\pi : L \text{ is strategic}\right) \\ &= \sigma \left|x_R^* - \left(x_L^* : L \text{ is sincere}\right)\right| + (1-\sigma)\left|x_R^* - \left(x_L^* : L \text{ is strategic}\right)\right| \end{aligned}$$

This variable $E\pi$ can be interpreted as the mean polarization in many elections of this type, perhaps across time or across different districts. As we can tell from its definition, the average polarization depends crucially on the equilibrium behavior of all the members of both parties, as summarized by the chosen platforms x_R^* and x_L^*. Theorem 5 reveals that the equilibrium value of $E\pi$ will ultimately depend only on the exogenous variable σ.

Theorem 5 *For a given value of σ, the average polarization in this type of elections is*

$$E\pi = \begin{cases} \sigma \text{ if } \sigma \in \left[0, \frac{1}{2}\right] \\ -2\sigma^2 + 6\sigma - 2 \text{ if } \sigma \in \left(\frac{1}{2}, 1\right] \end{cases}$$

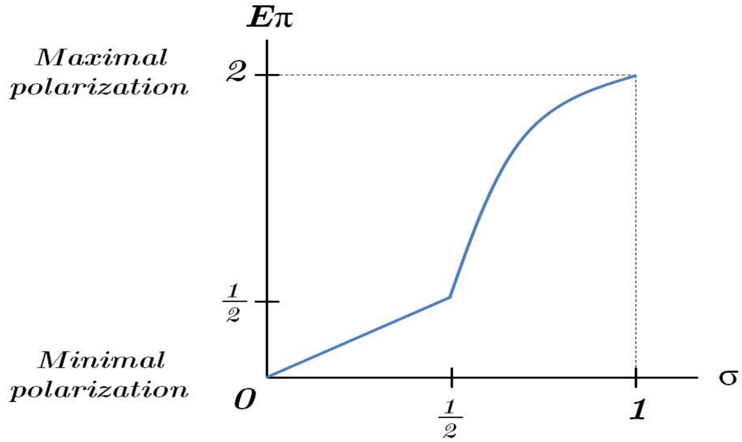
According to this theorem, the prior belief that party L will be sincere instead of strategic has a profound impact on the polarization that can be expected from this election. An increase in σ will increase $E\pi$ in the following three ways. (1) It increases the likelihood that the members of party L will sincerely adopt a completely extreme-left platform. (2) This expected behavior by L will in turn induce the members of R, who are fully rational and forward-looking, to respond by adopting a relatively more extreme-right platform. (3) This decision by R will in turn influence the members of L, if they turn out to be strategic, to adopt a relatively extreme-left platform in response. The overall effect of σ is described in corollary 3.

Corollary 3 *The average polarization, $E\pi$, is strictly increasing with the likelihood that the members of party L will be sincere, σ. It increases from its minimal value of $E\pi = 0$ to its maximal value of $E\pi = 2$ as σ increases from 0 to 1.*

Note that polarization can cover the whole range of feasible values as we vary σ. Importantly, this occurs without having to modify the ideal point of any voter or any party member. To convey this issue it is worth distinguishing two different types of ideological distance between political parties. One measure of polarization, which we are following in this paper, is the distance between the parties' platforms, as given by $|x_R^* - x_L^*|$. Another possible measure of polarization would be the distance between the true ideologies of the parties' median members, as given by $|r_M - l_M|$. In previous work I have called the former *platform polarization* and the latter *preference polarization*, and I have shown how these two apparently similar measures can actually behave very differently, even moving in opposite directions (Serra 2010). Corollary 3 tells us that *platform polarization* can dramatically increase in spite of *preference polarization* being constant. The profound effect of σ on $E\pi$ can be visualized in the following graph.

Figure 5

Average polarization as a function of party L's probability of being sincere



To better grasp the impact of these results, it will be useful to state separately what the outcomes of this election would be in two extreme situations, namely the cases where $\sigma = 0$ and where $\sigma = 1$. Corollary 4 states the outcome of the election if all members of both parties are known to be strategic.

Corollary 4 *If $\sigma = 0$, then $x_R^* = x_L^* = x^* = 0$ and $\pi = 0$.*

So in this case, as it turns out, both parties will converge completely to the ideal point of the median voter in the general electorate and there will be no polarization. Remarkably, this occurs in spite of both parties holding primary elections among members that might have ideologically extreme preferences; in fact, I have suggested that the ideal points of the median members of both parties, $r_M = 1$ and $l_M = -1$, can be interpreted as being quite extremist. This surprising result indicates that primary elections, even among party members with far-out ideologies, will not necessarily create any polarization – if those extreme members are rational, they will push their parties to adopt centrist platforms to prevent the rival party from winning.[22]

Corollary 5 states the outcome of the election if all members of party L are known to be sincere while all members of party R are known to be strategic. As it turns out, in this case party R will adopt the ideal point of its median member as if all its members behaved sincerely. Hence platforms will be as extreme, and polarization will be as large, as if all the members of both parties, including those in R, were sincere.

Corollary 5 *If $\sigma = 1$, then $x^* = x_R^* = 1$, $x_L^* = -1$, and $\pi = 2$.*

[22] This result reinforces what four other models with different assumptions have found: Kselman (2015b), Serra (2015), Woon (2016) and Serra (2017). In those four models, in spite of significant centrifugal forces, the rationality of primary voters may lead their parties to completely converge.

This is the clearest illustration of the contagion effect in this model. If party L is sure to diverge completely to the ideal point of its median member, party R will choose to diverge completely to the ideal point of its median member as well (or to an infinitesimally more moderate point to ensure victory). So the general electorate will face two extremist parties, which is a remarkable result. Both parties will behave as if they were completely sincere, diverging to their ideal points, even though only one of them is actually sincere while the other is a strategic party mimicking the sincerity of its rival. Thus polarization will be maximal.

Another interesting phenomenon is most clearly conveyed by the limit case in corollary 5. Suppose that, for any reason, the members of party L are particularly passionate about their views in this election. Perhaps they think their purism will serve to advance their views in the public debate. Accordingly, it has become clear to everyone else that they intend to vote sincerely in their primary, meaning that $\sigma = 1$. My results indicate that the outcome will be very unfortunate for the members of party L: they will allow party R to win with a very unfavorable platform for them. By myopically adopting an extreme-left platform reflecting their sincere preferences, they will allow the rival party to win the election with an extreme-right platform. This is a harsh price to pay for expressing their sincere feelings at the primary stage.

8 Conclusions: The expectation of sincere voting in primaries can be a source of polarization

The results in this paper have several empirical and theoretical ramifications that are worth discussing. In particular, they may provide relevant lessons for our understanding of ideological extremism and polarization as they relate to primary elections. Scholars and political observers have worried for a long time about the risks of ideological polarization, especially since several democracies, such as the American one, seem to be witnessing increasingly extremist political parties.[23] The early literature on spatial elections provided only limited insight into the nature of polarization. The foundational contributions such as Hotelling (1929), Downs (1957) and Black (1958) emphasized the strong pull of the median voter, especially in two-party systems. But a significant part of the modern literature in formal theory has been devoted to understanding the conditions that can lead to polarization in spatial models. My paper contributes to this effort with a diversity of equilibria that help explain a wide range of party positions. In fact, the results in this model provide a richer set of predictions than spatial models commonly do. Depending on the value of the exogenous parameters, my model finds equilibria of several sorts which include the following: complete convergence of both parties to the median voter (as in figure 2); complete divergence of both

[23] Fiorina, Abrams and Pope (2006); McCarty, Poole and Rosenthal (2006).

parties to their ideal points (in the extreme version of figure 3); a completely centrist party against a completely extremist one (as in figure 1); and two moderately partisan parties (as in figure 4).

A general goal of this paper was to scrutinize the effect that primary elections may have on ideological extremism. As was mentioned in the introduction, there exists a debate with opposite claims: some work has argued that primaries create significant polarization,[24] while other work has argued that any effect is not really significant.[25] My paper bridges the gap between these two sides by finding conditions for both complete convergence and large divergence in the location of two political parties, depending on exogenous circumstances. The model was geared to studying parties at their most micro level: the attitude and the decisions of their members. Parties were conceived as a collection of heterogeneous members with diverse ideal points in the left-right political spectrum. To fix ideas, I suggested that the location of the median member in each party should be thought of as quite extreme; this interpretation helps highlighting the difference between full convergence and large divergence. I assumed that each member, whatever her ideology, has an equal voice inside her party in a democratic process of nomination. Concretely, prior to the general election, each party designs its platform by consulting all its members in a closed primary election. The main interest was to study the effects of sophistication among party members, meaning whether they will behave sincerely or strategically in their primary election. In both scenarios of sincere behavior and strategic behavior, the median member was proved to play a pivotal role: her preferred platform will be a Condorcet winner able to beat any alternative platform. However, I found that a strategic median member wishes to push her party to the center more than a sincere one does.

Having theories comparing sincere and strategic behavior is germane, given that statistical studies have found both sorts of voting in primary elections (Hall and Snyder (2015)). Given that different statistical studies operationalize the concepts differently, it is worth trying to develop theoretical conceptualizations to help unify the literature. To distinguish these two concepts, I suggested to take a behavioral approach. In this theory, behaving strategically can be interpreted as being fully rational, while behaving sincerely can be interpreted as being boundedly rational. This distinction has not been studied systematically in the theoretical. Indeed, most previous models have treated all primary voters as being equally rational: either they are all strategic or they are all sincere. A few exceptions have

[24]Theoretical contributions include Owen and Grofman (2006); Jackson, Mathevet and Mattes (2007); Adams and Merrill (2008); Hirano, Snyder Ting (2009); Castanheira, Crutzen, Sahuguet (2010); Serra (2011); Snyder and Ting (2011); Hummel (2013); Adams and Merrill (2014); Hortala-Vallve and Mueller (2015); Kselman (2015a); Kselman (2015b); Ting, Snyder and Hirano (2015); Amorós, Puy and Martínez (2016); Grofman, Troumpounis and Xefteris (2016); Woon (2016). Empirical contributions include Gerber and Morton (1998), Burden (2001) and Burden (2004).

[25]Theoretical contributions include Kselman (2015b); Serra (2015); Woon (2016); and Serra (2017). Empirical contributions include Hirano, Snyder, Ansolabehere and Hansen (2010); Peress (2013); and McGhee, Masket, Shor, Rogers and McCarty (2014).

endeavored to make a dichotomous comparison between sincere vs. strategic behavior, but to my knowledge they have not included a continuous variable for parties' expected sophistication. In this paper, the main exogenous variable is the continuous probability that all the members of one of the parties will be sincere instead of strategic. This probability will turn out to have drastic effects.

First, I find that sincere members will want their party to adopt a more extremism platform than strategic members. Second, I find that such extremism due to the party's sincerity will induce the rival party to adopt an extremist platform as well. In fact, a higher probability of one party being sincere leads to higher extremism in the rival party, as if sincerity was somehow "contagious". Third, the total effect of one party's sincerity on both parties' extremism implies that polarization also increases. All this represents a theoretical explanation for polarization in two-party systems that had not been explicitly modeled before, which corresponds to placing blame for the extremism of both parties on the likelihood that one of them will be irrationally expressive in its primary. A fourth result with a more normative flavor is also worth mentioning. Let us picture the outcome when the members of one of the parties have made it fully clear that they intend to vote sincerely; perhaps they are doing so to promote their true preferences in the public conversation. Yet they will end up biasing the outcome exactly in the opposite direction, as they will enable the strategic rival to win with a radical platform of its own. In other words, by expressing their views sincerely they end up hurting those views.

I believe this model also offers some words of caution for statistical studies of elite polarization. According to these results, any empirical study of the effect of primary elections on polarization should control for the sophistication of party members. Independently of any other institutional or behavioral variable, the sophistication of party affiliates could have a strong effect. Moreover, this effect could be contagious: remarkably, whether the members of a party will vote sincerely for an extremist primary candidate might depend in large part on whether the members of the rival party are expected to do the same. So, the sophistication of party members of all parties should probably be controlled for. Finally, a broad goal of this paper is to convey that primary elections probably have a complex and contingent relationship to extremism, leading to polarization in some cases but not in others. The circumstances on which this depends are worth exploring in academic research.

A Appendix (to be posted online)

A.1 Proofs of all the results

A.1.1 Lemma 1

Proof. This follows directly from our definition of sincerity, whereby each member of L, including its median l_M, behaves as if she had the objective function in equation 1 regardless of party R's platform. For the median member, this implies seeking to maximize $U_{l_M}(x_L) = -(-1-x_L)^2$, which peaks at -1. ■

A.1.2 Theorem 1

Proof. All the party members of L are sincere, which means they will rank alternative platforms x_L according to their original utility function in equation 1 regardless of the platform adopted by the rival party R. These preferences are single-peaked and symmetric; in fact $U_{l_j}(x_L)$ is a quadratic function represented by an inverse parabola with a unique global and local maximum at the point $x = l_j$. Given that we assumed L to have an odd number n_L of party members, a unique median must exist in the distribution of ideal points. We have called l_M a member of L who holds this median ideal point, which we have adjusted to -1. All this allows us to invoke any standard version of Duncan Black's median voter theorem which guarantees that the ideal point of l_M can defeat any other proposed platform in a pairwise contest, and is therefore a Condorcet winner (Black 1958). I have assumed that a Condorcet winner, if it exists, will become the party's platform, so we conclude that $x_L^* = l_M = -1$. ■

A.1.3 Corollary 1

Proof. Recall that we have adjusted the ideal point of the median voter, M, to zero. I have also imposed the restriction that $x_R \in [0,1]$. From theorem 1 we know that $x_L^* = -1$. This implies that $0 \leq |x_R| \leq |x_L^*| = 1$. We can divide the possible values of x_R in two cases. In the case where $|x_R| < |x_L^*|$, party R is strictly more centrist than party L, so it will obviously win the election and implement its campaign platform. In the case where $|x_R| = |x_L^*|$, party R and party L are exactly equidistant from the median voter; but party R will still win the election due to the indifference assumption IA1 that I stated in the text, whereby M will vote for R if she is indifferent and L is sincere. So in all cases the winning platform will be $x^* = x_R$. The resulting payoffs to party members of R and L are derived by plugging x_R into their utility functions 3 and 2 respectively. ■

A.1.4 Lemma 2

Proof. The median member of party L would prefer her party's platform x_L to yield an electoral *outcome* as close as possible to her ideal point, $l_M = -1$. To determine the platform that would be optimal for her, we need to solve the following problem. Remember that in this equation, x_R is a fixed value chosen previously by R, whereas x_L is the variable that still needs to be chosen by L.

$$\max_{x_L} U_{l_M}(x^*) = -(-1 - x^*)^2 \text{ with } x^* = \begin{cases} x_R \text{ if } 0 \leq x_R < -x_L \\ x_L \text{ if } 0 \leq -x_L \leq x_R \end{cases}$$
$$\text{subject to } x_R \in [0,1] \text{ and } x_L \in [-1,0]$$

We start by studying the general situation where $x_R \neq 0$. There are two intervals to consider for x_L. For any value of x_L such that $x_R < -x_L$, party R will win the election and implement its platform x_R, which is strictly larger than zero; hence, $x^* = x_R > 0$. If on the contrary we have $0 \leq -x_L \leq x_R$, then party L will win the election and implement its platform x_L. Within this interval, given that $-1 = l_M \leq -x_R \leq x_L \leq 0$, the median member of L is better off with the leftmost platform platform, which is $x_L = -x_R$. (Remember from our indifference assumption IA2 that in this case where both parties are equidistant from the center, given that L is strategic, the median voter M will vote for L.) The maximum in the second interval yields a higher payoff than any value in the first interval, and hence is a global maximum. Therefore the platform that maximizes the payoff to the median member of L is $x_L = -x_R$.

To complete the proof, we need to study the limit case where $x_R = 0$. This case presents a problem in terms of optimization, because the platform implemented after the election will always be zero regardless of party L's decision, meaning that $x^* = 0$ for any x_L. So in principle, the median member should be indifferent between any conceivable platform x_L. However, to ensure that the median member's preferences are well behaved in a way that is continuous, I will make the following assumption: if two platforms x_L and x'_L yield the same winning platform x^*, making the median member indifferent, but one of them leads her party to win and the other one to lose, I will assume the median member will strictly prefer the platform where her party wins. This is equivalent to assuming lexicographic preferences that give the median member some gratification if her party wins, only if the implemented policy remains constant. Under this assumption, the median member will strictly prefer the platform $x_L = -x_R = 0$ to any other one, because then her party would win given our indifference assumption IA2. ∎

A.1.5 Theorem 2

Proof. The different members of party L would prefer their party's platform x_L to yield an electoral outcome as close as possible to their ideal points. They are driven by the maximization problem in equation 2. To be precise, the j^{th} member of L evaluates each possible platform x_L according to the following utility function. Remember that in this equation, x_R is a fixed value chosen previously by R, whereas x_L is a possible platform by L that is being evaluated by the party member. Remember also that the case $x_L = -x_R$ leads party L to win given our indifference assumption IA2.

$$U_{l_j}(x^*) = -(l_j - x^*)^2 \text{ with } x^* = \begin{cases} x_R \text{ if } 0 \leq x_R < -x_L \\ x_L \text{ if } 0 \leq -x_L \leq x_R \end{cases}$$

We would like to prove that the median member's optimal platform is a Condorcet winner. In other words we want to show that the platform $x_L = -x_R$ can beat any other proposal x'_L in a majority contest. For this we need to consider all possible intervals for the alternative x'_L compared to the median member's favorite $-x_R$.

We start by studying the general situation where $x_R \neq 0$. Consider the median member's preferred platform $-x_R$. There are two intervals to consider for the competing platform x'_L. On one hand, if party L chose a platform x'_L such that $x'_L < -x_R$, then the election winner would be party R who would implement its platform x_R. Remember we are assuming that $l_j \leq 0$ for all j. The minority of members whose ideal point is zero are indifferent between both options x_R and $-x_R$. But all the members such that $l_j < 0$ strictly prefer the outcome leading to $-x_R$ to the outcome leading to x_R. Given that this latter group includes the median member and everyone to her left, the platform $-x_R$ would receive a strict majority of votes against the competing platform x'_L.

On the other hand, if party L chose a platform x'_L such that $-x_R < x'_L$, then the election winner would be party L who would implement its platform x'_L. How many party members would actually prefer winning with x'_L rather than winning with $-x_R$? In fact, every member with an ideal point to the left of $-x_R$, which includes the median member and everyone to her left, prefers $-x_R$ to x'_L. Given that this represents more than half of the members of the party, $-x_R$ would beat x'_L in majority voting.

To complete the proof, we need to study the limit case where $x_R = 0$. This case presents a problem in terms of optimization, because the platform implemented after the election will always be zero regardless of party L's decision, meaning that $x^* = 0$ for any x_L. So in principle, all party members should be indifferent between any conceivable platform x_L. However, to ensure that the party member's preferences are well behaved in a way that is continuous, I will make for each of them the same assumption that I made above for the median member: if two platforms x_L and x'_L yield the same winning platform x^*, making the member indifferent, but one of them leads her party to win and the other one to lose, I will assume the member will strictly prefer the platform where her party wins. This is equivalent to assuming lexicographic preferences that give the member some gratification if her party wins, only if the implemented policy remains constant. Under this assumption, each party member, including the median, will strictly prefer the platform $x_L = -x_R = 0$ to any other one, because then their party would win given our indifference assumption IA2. Consequently the median's preferred platform of zero would receive all the votes.

In sum, $-x_R$ would beat every alternative x'_L and is thus a Condorcet winner. Throughout the paper I am assuming that a party will adopt a Condorcet winner as its platform, so we conclude that $x_L^* = -x_R$. ■

A.1.6 Corollary 2

Proof. For any platform x_R that party R may have adopted, we previously proved that party L will adopt the platform $x_L^* = -x_R$. Hence both parties will be equally extremist on opposite sides of the median voter. Given our indifference assumption IA2, L will win the election. This implies that the winning platform is $x^* = x_L^* = -x_R$. The resulting payoffs to each party member are derived by plugging $-x_R$ into their utility functions in equations 3 and 2. ■

A.1.7 Lemma 3

Proof. Our goal is to find the platform x_R that the median member r_M would prefer. For this we must solve the following maximization problem:

$$\max_{x_R} EU_{r_M}\left(x^* : x_R\right) = \sigma U_{r_M}\left(x^* : x_R,\ L \text{ is sincere}\right) + (1-\sigma)\, U_{r_M}\left(x^* : x_R,\ L \text{ is strategic}\right)$$
$$\text{s.t. } x_R \in [0,1] \text{ and } \sigma \in [0,1]$$

Plugging into equation 4 the ideal point of the median, $r_M = 1$, we obtain:
$EU_{r_M}\left(x^* : x_R\right) = -\sigma\left(1-x_R\right)^2 - (1-\sigma)\left(1+x_R\right)^2 = -x_R^2 + 4x_R\sigma - 2x_R - 1$
Differentiating this expression we obtain:
$\frac{\partial EU_{r_M}(x^*)}{\partial x_R} = \frac{\partial}{\partial x_R}\left(-x_R^2 + 4x_R\sigma - 2x_R - 1\right) = -2x_R + 4\sigma - 2$
The first-order conditions imply:
$\frac{\partial EU_{r_M}(x^*)}{\partial x_R} \geq 0 \Leftrightarrow -2x_R + 4\sigma - 2 \geq 0 \Leftrightarrow 2\sigma - 1 \geq x_R$, so the unique critical point is $x_R = 2\sigma - 1$.

The second-order conditions give:
$\frac{\partial^2 EU_{r_M}(x^*)}{\partial x_R^2} = \frac{\partial}{\partial x_R}\left(-2x_R + 4\sigma - 2\right) = -2$, which is negative, so the critical point, if it is reached, must be a maximum.

However, this critical point does not always belong to the admissible interval $x_R \in [0,1]$. To see this, we must divide the values of σ in different ranges, remembering that $\sigma \in [0,1]$.

If $\sigma > \frac{1}{2}$ then $2\sigma - 1 \in (0, 1]$, so the first-order conditions are met exactly at the interior point $x_R = 2\sigma - 1$, which is the argmax in the admissible range.

However, if $\sigma < \frac{1}{2}$ then $2\sigma - 1 < 0$, so the first-order conditions are never met because it is impossible to have $\frac{\partial EU_{r_M}(x^*)}{\partial x_R} = 0$ for any positive value of x_R. On the contrary, we have $\frac{\partial EU_{r_M}(x^*)}{\partial x_R} < 0$ for any $x_R \geq 0$. This implies that $EU_{r_M}(x^* : x_R)$ is decreasing in x_R, which means there is a corner solution whereby the maximum is obtained at the minimum allowed value of x_R, namely $x_R = 0$, which is the argmax in the admissible range.

Finally, if $\sigma = \frac{1}{2}$ then $2\sigma - 1 = 0$, so the first-order conditions are met exactly at the corner point $x_R = 0$, which is the argmax in the admissible range. ■

A.1.8 Theorem 3

Proof. Each different member of party R would like her party's platform x_R to yield the highest possible expected utility for her. To be precise, the i^{th} member of R evaluates each possible platform x_R according to the expected utility in equation 4, which depends on her ideal point r_i. Suppose the i^{th} member of party R is comparing two options, labelled x_R and x'_R. What conditions must be true for this member to strictly prefer x_R to x'_R? With some algebra, we can find that:

$EU_{r_i}(x^* : x'_R) < EU_{r_i}(x^* : x_R) \Leftrightarrow -\sigma (r_i - x'_R)^2 - (1 - \sigma)(r_i + x'_R)^2 < -\sigma (r_i - x_R)^2 - (1 - \sigma)(r_i + x_R)^2 \Leftrightarrow 0 < (x'_R - x_R)[x_R + x'_R + 2r_i(1 - 2\sigma)]$

We would like to prove that the median member's optimal platform for each σ is a Condorcet winner. In other words we want to show that the platform x_R, as defined in lemma 3, can beat any other platform proposal x'_R in a majority contest. Note that we need to consider the entire possible range for σ. In each case, we want to prove that the optimal platform for the median is strictly preferred to any other alternative by a strict majority of members. So we would like to know which are the party members r_i for which the expected utility with x_R, as defined in lemma 3, is strictly higher than with an alternative x'_R. For this, we must solve the inequality above for r_i. We divide the proof according to different intervals for σ.

- Case $\sigma \in \left[0, \frac{1}{2}\right]$

In this case, the optimal platform x_R for the median member r_M is zero, according to lemma 3. Then theorem 3 claims that this is a Condorcet winner. So we must prove that a strict majority of members r_i have a strictly higher expected utility from the platform $x_R = 0$ than an alternative platform x'_R. Note that any alternative platform must be strictly larger than zero, meaning $x'_R > 0$, due to our assumption that any platform by R must fall between zero and one. How many voters would prefer the median's favorite, $x_R = 0$, to an alternative $x'_R > 0$? Given the model's assumptions, we can prove the following equivalences:

$EU_{r_i}(x^* : x'_R) < EU_{r_i}(x^* : x_R = 0)$

$\Leftrightarrow 0 < (x'_R - x_R)[x_R + x'_R + 2r_i(1-2\sigma)]$ from the general result found above

$\Leftrightarrow 0 < x_R + x'_R + 2r_i(1-2\sigma)$ given that $(x'_R - x_R) > 0$ because $x'_R > x_R$

$\Leftrightarrow 0 < x'_R + 2r_i(1-2\sigma)$ given that $x_R = 0$

But this last inequality is always true for any party member because all the terms are positive, and one is strictly positive. To be precise, we have $x'_R > 0$; $r_i \geq 0$; and $0 \leq 1-2\sigma$ in this interval for σ. We conclude that for any ideal point, r_i, the i^{th} member of R strictly prefers the platform $x_R = 0$ to the alternative $x'_R > 0$. Therefore zero is a Condorcet winner and will be adopted as a platform in the primary.

- Case $\sigma \in \left(\frac{1}{2}, 1\right]$

In this case, the optimal platform x_R for the median member r_M is $2\sigma - 1$, according to lemma 3. Then theorem 3 claims that this is a Condorcet winner. So we must prove that a strict majority of members r_i have a strictly higher expected utility from the platform $2\sigma - 1$ than an alternative platform x'_R. How many voters would prefer the median's favorite, $x_R = 2\sigma - 1$, to an alternative x'_R? We need to consider two ranges for x'_R, namely the intervals with more extremist or more centrist alternatives to x_R.

First, consider $x'_R > x_R$, meaning $x'_R > 2\sigma - 1$. Given the model's assumptions, we can prove the following equivalences:

$EU_{r_i}(x^* : x'_R) < EU_{r_i}(x^* : x_R = 2\sigma - 1)$

$\Leftrightarrow 0 < (x'_R - x_R)[x_R + x'_R + 2r_i(1-2\sigma)]$ from the general result found above

$\Leftrightarrow 0 < x_R + x'_R + 2r_i(1-2\sigma)$ given that $(x'_R - x_R) > 0$ because $x'_R > x_R$

$\Leftrightarrow 0 < (2\sigma - 1) + x'_R + 2r_i(1-2\sigma)$ replacing x_R by its value $2\sigma - 1$

$\Leftrightarrow 2r_i(2\sigma - 1) < (2\sigma - 1) + x'_R$

$\Leftrightarrow r_i < \frac{1}{2} + \frac{x'_R}{2(2\sigma-1)}$ using the fact that $2\sigma - 1 > 0$ in this case where $\sigma > \frac{1}{2}$

$\Leftrightarrow r_i < \frac{1}{2} + \frac{x'_R}{2(2\sigma-1)} + \frac{1}{2} - \frac{1}{2}$

$\Leftrightarrow r_i < 1 + \frac{x'_R - (2\sigma-1)}{2(2\sigma-1)}$

$\Leftrightarrow r_i < r_M + \frac{x'_R - (2\sigma-1)}{2(2\sigma-1)}$ recalling that the ideal point of the median party member, r_M, is 1.

Here we must note that $\frac{x'_R - (2\sigma-1)}{2(2\sigma-1)} > 0$ because $x'_R > (2\sigma - 1)$. Therefore, this condition is met at least by the median party member r_M and every party member r_i with a lower ideal point. Thus any party member with an ideal point $r_i \leq 1$, strictly prefers the platform $x_R = 2\sigma - 1$ to the alternative $x'_R > 2\sigma - 1$. We conclude that strictly more than half of the party members, including the median and everyone to her left, prefers the median's favorite platform to x'_R.

Second, consider $x'_R < x_R$, meaning $x'_R < 2\sigma - 1$. Given the model's assumptions, we can prove the following equivalences:

$EU_{r_i}(x^* : x'_R) < EU_{r_i}(x^* : x_R = 2\sigma - 1)$

$\Leftrightarrow 0 < (x'_R - x_R)[x_R + x'_R + 2r_i(1-2\sigma)]$ from the general result above

$\Leftrightarrow 0 > x_R + x'_R + 2r_i\,(1-2\sigma)$ given that $(x'_R - x_R) < 0$ because $x'_R < x_R$
$\Leftrightarrow 0 > (2\sigma - 1) + x'_R + 2r_i\,(1-2\sigma)$ replacing x_R by its value $2\sigma - 1$
$\Leftrightarrow 2r_i\,(2\sigma-1) > (2\sigma-1) + x'_R$
$\Leftrightarrow r_i > \frac{1}{2} + \frac{x'_R}{2(2\sigma-1)}$ using the fact that $2\sigma - 1 > 0$ in this case where $\sigma > \frac{1}{2}$
$\Leftrightarrow r_i > \frac{1}{2} + \frac{x'_R}{2(2\sigma-1)} + \frac{1}{2} - \frac{1}{2}$
$\Leftrightarrow r_i > 1 - \frac{(2\sigma-1)-x'_R}{2(2\sigma-1)}$
$\Leftrightarrow r_i > r_M - \frac{(2\sigma-1)-x'_R}{2(2\sigma-1)}$ recalling that the ideal point of the median party member, r_M, is 1.

Here we must note that $\frac{(2\sigma-1)-x'_R}{2(2\sigma-1)} > 0$ because $x'_R < (2\sigma-1)$. Therefore, this condition is met at least by the median party member r_M and every party member r_i with a higher ideal point. Thus any party member with an ideal point $r_i \geq 1$, strictly prefers the platform $x_R = 2\sigma - 1$ to the alternative $x'_R < 2\sigma - 1$. We conclude that strictly more than half of the party members, including the median and everyone to her left, prefers the median's favorite platform to x'_R.

In sum, the inequality $EU_{r_i}\,(x^* : x'_R) < EU_{r_i}\,(x^* : x_R = 2\sigma - 1)$ is true for a strict majority of party members for any $x'_R \in \mathbb{R}$. Therefore, the median's favorite, $2\sigma - 1$, can beat any alternative in a pairwise majority vote. It is a Condorcet winner and it will be adopted as a platform in the primary. ■

A.1.9 Theorem 4

Proof. We need to solve four different scenarios, as they are described in Table 1.

- Case $\sigma \in \left[0, \frac{1}{2}\right]$

We simply compute the values from previous theorems for the case where $\sigma \in \left[0, \frac{1}{2}\right]$. At stage 1 of the game, we know from theorem 3 that $x^*_R = 0$. At stage 2 there are two possibilities: party L will be either sincere or strategic.

1. L is sincere

   If L turns out to be sincere, at stage 3, L's platform will be determined by theorem 1, meaning that $x^*_L = -1$. Then, at stage 4, we know from corollary 1 that party R will win the election and hence $x^* = x^*_R = 0$.

2. L is strategic

   If L turns out to be strategic, at stage 3, L's platform will be determined by theorem 2, meaning that $x^*_L = -x^*_R = 0$. Then, at stage 4, we know from corollary 2 that party L will win the election and hence $x^* = x^*_L = 0$.

- Case $\sigma \in \left(\frac{1}{2}, 1\right]$

We simply compute the values from previous theorems for the case where $\sigma \in \left(\frac{1}{2}, 1\right]$. At stage 1 of the game, we know from theorem 3 that $x_R^* = 2\sigma - 1$. At stage 2 there are two possibilities: party L will be either sincere or strategic.

1. L is sincere

   If L turns out to be sincere, at stage 3, L's platform will be determined by theorem 1, meaning that $x_L^* = -1$. Then, at stage 4, we know from corollary 1 that party R will win the election and hence $x^* = x_R^* = 2\sigma - 1$.

2. L is strategic

   If L turns out to be strategic, at stage 3, L's platform will be determined by theorem 2, meaning that $x_L^* = -x_R^* = -2\sigma + 1$. Then, at stage 4, we know from corollary 2 that party L will win the election and hence $x^* = x_L^* = -2\sigma + 1$.

■

A.1.10 Theorem 5

Proof. We will divide the possible values of the exogenous variable in two different intervals: $\sigma \in \left[0, \frac{1}{2}\right]$ and $\sigma \in \left(\frac{1}{2}, 1\right]$. In each case, we will appeal to theorems 1, 2 and 3 for the values of x_L^* and x_R^* in equilibrium.

If $\sigma \in \left[0, \frac{1}{2}\right]$ we know the following: $x_R^* = 0$; if L is sincere then $x_L^* = -1$; if L is strategic then $x_L^* = -x_R^* = 0$. So the average polarization, from its definition in the text, can be calculated to be:

$E\pi = \sigma \left| x_R^* - (x_L^* : L \text{ is sincere}) \right| + (1-\sigma) \left| x_R^* - (x_L^* : L \text{ is strategic}) \right| = \sigma \left| 0 - (-1) \right| + (1-\sigma) \left| 0 - 0 \right| = \sigma$

If $\sigma \in \left(\frac{1}{2}, 1\right]$ we know the following: $x_R^* = 2\sigma - 1$; if L is sincere then $x_L^* = -1$; if L is strategic then $x_L^* = -x_R^* = 1 - 2\sigma$. So the average polarization, from its definition in the text, can be calculated to be:

$E\pi = \sigma \left| x_R^* - x_L^* : L \text{ is sincere} \right| + (1-\sigma) \left| x_R^* - x_L^* : L \text{ is strategic} \right| = \sigma \left| (2\sigma - 1) - (-1) \right| + (1-\sigma) \left| (2\sigma - 1) - (1 - 2\sigma) \right| = \sigma (2\sigma) + (1-\sigma)(4\sigma - 2) = -2\sigma^2 + 6\sigma - 2$ ■

A.1.11 Corollary 3

Proof. We will divide the possible values of the exogenous variable in two different intervals: $\sigma \in \left[0, \frac{1}{2}\right]$ and $\sigma \in \left(\frac{1}{2}, 1\right]$. In each case, we will prove that $E\pi$ is strictly increasing by showing that its derivative with respect to σ is strictly positive. We appeal to theorem 5 for the value of $E\pi$ as a function of σ.

If $\sigma \in \left[0, \frac{1}{2}\right]$, then $E\pi = \sigma$. In this interval, we thus have $\frac{\partial E\pi}{\partial \sigma} = 1 > 0$

If $\sigma \in \left(\frac{1}{2}, 1\right]$, then $E\pi = -2\sigma^2 + 6\sigma - 2$. In this interval, we thus have $\frac{\partial E\pi}{\partial \sigma} = 6 - 4\sigma$. This derivative will be strictly positive if and only if $\frac{\partial E\pi}{\partial \sigma} > 0 \Leftrightarrow 6 - 4\sigma > 0 \Leftrightarrow \sigma < 1.5$ which is always fulfilled because $\sigma \leq 1$.

Finally, from theorem 5, we calculate the following values

$$E\pi = \begin{cases} 0 \text{ if } \sigma = 0 \\ -2\left(1\right)^2 + 6\left(1\right) - 2 = 2 \text{ if } \sigma = 1 \end{cases}$$

■

A.1.12 Corollary 4

Proof. We simply compute the values from previous theorems for the case where $\sigma = 0$. At stage 1 of the game, we know from theorem 3 that $x_R^* = 0$. At stage 2, party L will be strategic for sure given that $\sigma = 0$. Hence at stage 3, we know with certainty that L's platform will be determined by theorem 2, meaning that $x_L^* = -\ x_R^* = 0$. Finally, at stage 4, we know from theorem 4 that $x^* = 0$. Since in this case there is no uncertainty, the actual polarization π will be the same as the average polarization $E\pi$, which we know from theorem 5 to be 0. ■

A.1.13 Corollary 5

Proof. We simply compute the values from previous theorems for the case where $\sigma = 1$. At stage 1 of the game, we know from theorem 3 that $x_R^* = 1$. At stage 2, party L will be sincere for sure given that $\sigma = 1$. Hence at stage 3, we know with certainty that L's platform will be determined by theorem 1, meaning that $x_L^* = 1$. Finally, at stage 4, we know from theorem 4 that $x^* = 1$. Since in this case there is no uncertainty, the actual polarization π will be the same as the average polarization $E\pi$, which we know from theorem 5 to be $-2\left(1\right)^2 + 6\left(1\right) - 2 = 2$. ■

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