Número 295

Should We Expect Primary Elections to Create Polarization?

A Robust Median Voter Theorem with Rational Parties

GILLES SERRA

Octubre 2017

CENTRO DE INVESTIGACIÓN Y DOCENCIA ECONÓMICAS

Advertencia

Los Documentos de Trabajo del CIDE son una herramienta para fomentar la discusión entre las comunidades académicas. A partir de la difusión, en este formato, de los avances de investigación se busca que los autores puedan recibir comentarios y retroalimentación de sus pares nacionales e internacionales en un estado aún temprano de la investigación.

De acuerdo con esta práctica internacional congruente con el trabajo académico contemporáneo, muchos de estos documentos buscan convertirse posteriormente en una publicación formal, como libro, capítulo de libro o artículo en revista especializada.

Carretera México Toluca 3655, Col. Lomas de Santa Fe, 01210,
Álvaro Obregón, Ciudad de México, México.
www.cide.edu

www.LibreriaCide.com

Oficina de Coordinación Editorial
editorial@cide.edu
Tel. 5727 9800

Acknowledgements

Gilles Serra is assistant professor at the Division of Political Studies at CIDE. He would like to thank his research assistant Gerardo Manrique De Lara Ruiz for great help.

Email: gilles.serra@cide.edu

Personal page: http://www.investigadores.cide.edu/gilles.serra

Abstract

A core concern about the use of primary elections is whether they induce political parties to adopt extremist ideological positions. Many scholars and pundits have blamed primary elections as a source of polarization, but recent statistical studies from the United States have not found such effect. So this essay undertakes a theoretical exploration of the issue. I develop a basic Downsian model adding a nomination stage where candidates need to compete within their parties before being able to run for office. I derive a median-voter theorem whereby all candidates are expected to converge to the center of the political spectrum. One of the reasons is the rationality of primary voters: even if they have extremist ideal points, party members understand the importance of voting strategically by choosing a moderate candidate who can prevent the other party from winning. The theorem is robust to several extensions, such as varying the risk aversion of voters and allowing candidates to care about winning the nomination independently of winning the general election. A conclusion is that primary elections are not sufficient to create polarization by themselves. Rather, for candidates to diverge to the extremes, other behavioral or institutional features must interact with primaries.

Keywords: Primary elections, polarization, sincere voting, strategic voting

Should we expect primary elections to create polarization? A robust median voter theorem with rational parties

Gilles Serra*

October 15, 2017

Abstract

A core concern about the use of primary elections is whether they induce political parties to adopt extremist ideological positions. Many scholars and pundits have blamed primary elections as a source of polarization, but recent statistical studies from the United States have not found such effect. So this chapter undertakes a theoretical exploration of the issue. I develop a basic Downsian model adding a nomination stage where candidates need to compete within their parties before being able to run for office. I derive a median-voter theorem whereby all candidates are expected to converge to the center of the political spectrum. One of the reasons is the rationality of primary voters: even if they have extremist ideal points, party members understand the importance of voting strategically by choosing a moderate candidate who can prevent the other party from winning. The theorem is robust to several extensions, such as varying the risk aversion of voters and allowing candidates to care about winning the nomination independently of winning the general election. A conclusion is that primary elections are not sufficient to create polarization by themselves. Rather, for candidates to diverge to the extremes, other behavioral or institutional features must interact with primaries.

*Department of Political Science, Center for Economics Research and Teaching (CIDE), Mexico City, Mexico, gilles.serra@cide.edu

Resumen

Una preocupación central acerca del uso de elecciones primarias es si inducen a los partidos políticos a adoptar posiciones ideológicas extremistas. Muchos académicos y observadores han culpado a las elecciones primarias de ser una fuente de polarización, pero algunos estudios estadísticos recientes de los Estados Unidos han encontrado que existe dicho efecto. Así que este trabajo lleva a cabo una exploración teórica de la cuestión. Desarrollo un modelo basado en el canónico modelo espacial de Anthony Downs, al cual agrego una etapa de nominación en la que los candidatos necesitan competir dentro de sus partidos antes de buscar un cargo. Derivo un teorema del votante mediano mediante que predice que todos los candidatos van a converger al centro del espectro político. Una de las razones es la racionalidad de los votantes en la primaria: aunque estos tengan puntos ideales extremos, los miembros de los partidos entienden la importancia votar estratégicamente escogiendo un candidato moderato que pueda evitar que el otro partido gane. El teorema es robusto a varias extensiones, tales como variar la aversión al riesgo de los votante y permitir que los candidatos deseen ganar la nominación independientemente de su deseo por ganar la elección general. Una conclusión es que las elecciones primarias no son suficientes para crear polarización por sí mismas. Más bien, para que los candidatos diverjan a los extremos, otras características conductuales o institucionales deben interactuar con las primarias.

Palabras clave: Elecciones primarias, polarización, voto sincero, voto estratégico

1 The conjectured impact of primary elections on polarization

The introduction of primary elections is often presumed to carry important policy consequences. Every political party needs a procedure to nominate the person it will postulate for office at an upcoming election. Such a procedure is sometimes called a candidate-selection method (CSM); and primary elections are only one of many possible methods. Historically, parties across the world have employed a diverse array of nomination processes such as delegate conventions and elite appointments; and only in recent times have primaries become more frequent.[1] In the United States, for example, the introduction of the direct primary is associated with the Progressive era, roughly between 1890 and 1920. A number of legal reforms during this period were geared to disempowering party bosses: primary elections were conceived as a way of transferring the responsibility to nominate candidates from a few hundred convention delegates to thousands of party members.[2] Among other goals, the reformers that advocate for primary elections in their countries are usually attempting to make parties more responsive to their rank-and-file members. Internal democracy is thus hailed as major benefit of introducing primaries.[3]

Notwithstanding this valuable benefit, several observers have worried about the social costs they see in primary elections. Probably the most-often mentioned cost of introducing primary elections is ideological polarization. Indeed, many scholars and pundits, especially in America, have conjectured that such CSM leads to the extremism of candidates' platforms. To be sure, some persuasive arguments can be made to expect such a polarizing effect, at least theoretically. A common claim is that primary voters have more extremist preferences than the general population, especially in *closed* primaries that only include registered adherents as compared to *open* primaries that include any citizen. This supposedly gives an advantage to extremist primary contenders, and forces moderate primary contenders to diverge away from the ideological center. This popular claim remained speculative for many decades, until

[1] For a recent survey of nomination procedures around the world, see Sandri and Seddone (2015).
[2] A new account of the Progressive era comes in Masket (2016).
[3] A review of the positive and negative consequences of primaries that have been found in the literature, along with the causes for the introduction of primaries around the world, can be found in Serra (2018).

it started being tested by a series of increasingly sophisticated empirical articles in academic journals.

One of the pioneering papers testing this claim was Gerber and Morton (1998). The goal of their paper was to measure how different types of nomination procedure for congressional positions led to selecting candidates with different ideologies. They were particularly interested in whether the extremism of the selectorate led to the extremism of the nominees. They speculated that closed primaries have a polarizing effect:

> "To the extent that members of the parties are ideologically distinct, we therefore expect the ideal point of the primary electorate median voter in closed primaries to reflect the ideological positions of the party's elite and to diverge substantially from the ideal point of the general electorate median voter. (...) The main hypothesis is that closed primaries will produce general election winners whose policy positions diverge substantially from their district's general election median voter." (pp. 311-312)

Their statistical results supported this hypothesis by finding that representatives from closed primary systems were more extremists than representatives from other CSMs with more moderate selectorates (such as semi-closed, open, non-partisan, and blanket primary systems). Other early studies seemed to confirm this finding which, in turn, encouraged theories to make this prediction. Given the traditional belief that primaries create polarization, along with a first wave of empirical papers that seemed to confirm this view, it is not surprising that a significant number of formal models have been developed to be consistent with such claim.[4]

While theoretically compelling, formal models predicting that primaries polarize candidates are at odds with the new empirical evidence. Indeed, some recent statistical studies have been casting doubt on this view, finding instead that closed primary elections have no

[4]Models of primaries predicting some sort of divergence include, among others, Jackson, Mathevet and Mattes (2007); Adams and Merrill (2008); Serra (2011); Snyder and Ting (2011); Hummel (2013); Hortala-Vallve and Mueller (2015); Kselman (2015); Amorós, Puy and Martínez (2016); Woon (2016); Grofman, Troumpounis and Xefteris (2016); and Serra (2017).

effect, or a negligible one, on the extremism of candidates.[5] For example, McGhee, Masket, Shor, Rogers and McCarty (2014) use state-level data to gauge the effect of primary openness on the polarization of local legislatures. For each state, the authors measure the degree to which primary selectorates are inclusive rather than exclusive (meaning the degree to which primaries are open rather than closed); and they marry this data to estimations of the ideal points of state legislators. Surprisingly they find their estimated effects to be rarely robust, meaning there is little effect of the type of CSM. In some of their specifications, there is a statistically significant effect but it goes in the opposite direction of the one expected: primaries that are more closed by virtue of having more exclusive selectorates (and which therefore should have more extremist voters) end up electing legislators that are more moderate.

These new statistical studies pose a challenge from the theoretical point of view. With several formal models of primaries predicting polarization, why are the newest empirical studies not finding it? One possible interpretation is that primaries have in fact a contingent effect, leading to polarization in some contexts but not in others. If so, is it possible to build a formal model where primaries do not lead to extremism? Such a model would allow us to compare its assumptions with the assumptions of models where primaries lead to extremism. In turn, this would help understand where extremism really comes from in those other theories, thus shedding light on this controversy.

Such is the goal of this chapter where I develop a model to investigate the effect that we should expect from primary elections on policy polarization. The model is purposely simple: to the well-known linear model developed by Anthony Downs (1957), I only add a nomination stage with two political parties where candidates need to compete before being able to run for office. The model explicitly incorporates a number of features that are considered *centrifugal*, meaning that they create incentives for candidates to diverge away from the center. First, I assume that the two parties have extremist ideologies on opposite sides of the median voter. Second, neither party cares about winning the election *per se*, but rather they care only about the policy implemented by the candidate who wins the

[5] Statistical studies claiming there is little-to-no effect of primaries on divergence include Hirano, Snyder, Ansolabehere and Hansen (2010); Peress (2013); and McGhee, Masket, Shor, Rogers and McCarty (2014).

election. Third, once a candidate promises a policy to her party in the primary, this promise becomes binding in the general election as well. Fourth, while candidates receive a payoff if they win the general election, I will assume that they also receive an independent payoff from winning the nomination in their parties. And fifth, I study the case where parties are risk-seekers, meaning that among two candidates yielding the same expected policy, parties prefer nominating a risky extremist rather than a riskless centrist. All these assumptions are stacking the deck in favor of obtaining extremism – and yet the model does not find any. In line with the most recent empirical literature, I find that closed primaries do not induce candidates to diverge from each other at all. One of the reasons is the rationality of primary voters: even if they have extremist ideal points, party members understand the importance of voting strategically by choosing a moderate candidate who can prevent the other party from winning. Hence my model underlines an understudied feature of primary elections that might have a profound effect on their outcomes: the rationality of party members whereby they vote strategically rather than sincerely.[6] It turns out that this assumption alone, at least in a bare-bones model, is enough to induce all primary candidates from both parties to converge completely to the median voter's ideal point.

In short, this chapter provides a median-voter theorem with competitive nominations. The result can be seen as "robust" in the sense that it generalizes a previous theory. In Serra (2015), I derived a median-voter theorem with nominations which included the first three of the conditions mentioned above: parties with extremist ideal points; parties that care about influencing policy rather than winning the election; and primary platforms that are binding in the general election. The model in this chapter extends the analysis by adding the last two conditions: candidates who value obtaining the nomination independently of winning the election; and parties who are risk loving. Given that the result remains unchanged, the theorem here can be considered robust.[7]

[6] It should be noted that several empirical studies of political behavior have found strategic voting in primary elections. See Hall and Snyder (2015) and the citations therein.

[7] A couple of recent models with different setups from mine have also proved theoretically that primaries do not necessarily lead to polarization: Kselman (2015) and Woon (2016) derived conditions for complete convergence in spite of primaries.

2 Structure of the election

2.1 Timing

The election is modeled as a three-stage game between voters, parties and candidates. The three stages correspond to the *platform announcements by candidates*, the *nominations by parties* and the *general election by voters*, in this sequence. The goal of the election is to decide a policy to be implemented. Each policy platform is represented by a point x in the policy space $\mathbb{R}$, where $\mathbb{R}$ is the real line. There are two parties, labeled L for the left-wing party and R for the right-wing party. Each party needs to nominate a candidate for office among those who are competing inside the party, often referred to as *precandidates*. There are four such precandidates, which are labeled l_1, l_2 for those in party L and r_1, r_2 for those in party R. The only distinguishable characteristic of each candidate is the policy platform she adopts. Hence, throughout the chapter I will make no distinction between a candidate and her platform, referring to l_1, l_2, r_1, r_2, when talking either about the candidates' platforms or the candidates themselves.

In the first stage, the four candidates announce their platforms simultaneously. A candidate's strategy consists on announcing a policy platform in $\mathbb{R}$. We denote a profile of candidate strategies by S_c, with $S_c = (l_1, l_2, r_1, r_2)$. The platform that a candidate adopts during the nomination process represents a binding commitment: it will become her platform for the general election at the subsequent stage, and it will be the policy she implements if she is elected.[8]

In the second stage, for a given set of platforms announcements S_c, party L must choose a candidate l_i and party R must choose a candidate r_j to compete against each other in the general election. So after observing the profile (l_1, l_2, r_1, r_2), party L nominates either l_1 or l_2 while R nominates either r_1 or r_2. Both parties nominate their candidates simultaneously.[9]

[8] I am thus discarding the possibility of flip-flopping during the election season. One way to justify this assumption is that, in this election, flip-flopping would hurt the candidate's credibility so much that it would never be an optimal strategy. This should actually stack the deck in favor of divergence, given that any promise made to primary voters will be "sticky" throughout the election.

[9] Table 1 in the appendix contains an exhaustive list of all the possible configurations of platforms that candidates my adopt.

We denote by S_L the strategy of L and by S_R the strategy of R. A party's strategy consists of a complete plan of action contingent on every possible situation in which it might be called upon to act. In the present context this implies specifying an action for each possible configuration of platforms that it may observe. Since every set of candidate platforms $S_c = (l_1, l_2, r_1, r_2)$ forms a subgame of this game, a strategy for a party specifies a nomination for each of those configurations. Therefore both S_L and S_R are complete mappings from $\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ into $\mathbb{R}$.

Lastly, in the third stage, voters elect a party to take office. We will assume that a median voter in the general electorate exists whose decision is pivotal. We call the median voter M.

The timing of the game is summarized as follows:

1. *Platform announcements*: All 4 candidates, r_1, r_2 in party R and l_1, l_2 in party L, announce their policy platforms simultaneously.

2. *Nominations*: Both parties, R and L, choose their nominees simultaneously.

3. *General election*: The median voter in the electorate, M, elects one of the two parties.

This being the basic structure of the election, here are details about the preferences of voters, parties and candidates.

2.2 Voters' preferences

We will assume voters' preferences to be single-peaked and linear with ideal points in $\mathbb{R}$. There exists a median voter called M, whose preferences are decisive.[10] Her ideal point is known with certainty to everyone, and we normalize it to zero. M's utility function is thus given by

$$U_M(x) = -|x|$$

Given such preferences, the behavior of voters is trivial when they have to choose between the two parties: they will always vote for the one whose candidate has a platform closest

[10] All the results would hold for any symmetric and single-peaked utility function for voters. The linear one is used as an illustration.

to their ideal points, so the party whose candidate announced a platform closest to median voter's ideal point will be elected. In other words, the party closest to zero will win. If the platforms of parties yield the same utility to M, then she will randomize her vote such that either party will win the election with equal probability. Hence if party R and party L were equidistant from zero, they would tie having each a $\frac{1}{2}$ probability of winning.

2.3 Risk-averse, risk-seeking or risk-neutral parties

Parties L and R care about the policy implemented by the elected official. In other words, they are *policy-motivated* meaning that they have ideal points over policy.[11] The only action taken by parties is to nominate their candidates for the general election. Each party holds a closed primary election where all its members vote democratically to choose the nominee. Here I will eschew modeling explicitly the thousands, sometimes millions, of party sympathizers that attend a primary election. Instead I will assume that each party has a median member whose preferences will be decisive in the primary. In parallel research I have proved that, as long as all primary voters have single-peaked preferences, the platform preferred by the median party member is a Condorcet winner – and this will be true even when all party members are strategic rather than sincere (Serra 2017). An implication of this result is that a party can reasonably be treated as a unitary actor behaving strategically based on the ideal point of its median member.

Both parties are rational and forward looking, meaning they will try anticipating the other players' reactions. Using the jargon in political science, we would say that parties are *strategic* rather than *sincere*. As a consequence, a party will not blindly nominate the candidate closest to its ideal point. On the contrary, a party will often be willing to nominate a moderate candidate if she has a higher chance of winning the election. In essence, each party must find the candidate that best balances its desire for a partisan platform with its fear of letting the other party win. It will do so while taking into account the candidate that is expected to be nominated by the rival party, meaning that the simultaneous nominations made by parties L and R need to form a Nash equilibrium.

[11] This follows the tradition of Wittman (1973) and Calvert (1985).

I will assume the ideal points of both parties to be on opposite sides of the median voter, such that we genuinely have a left-wing party and a right-wing party. To simplify the presentation, we will assume that L's ideal point is -1 while R's ideal point is 1. For interpretation purposes, we should think of the locations -1 and 1 as being quite extreme on the left and the right of the political spectrum.[12] Parties have single-peaked preferences represented by the following utility functions:

$$U_R(x) = -|1-x|^a$$
$$U_L(x) = -|-1-x|^a$$
$$\text{with } a > 0$$

In the equations above, the parameter a represents each party's *attitude toward risk* as represented by the concavity or convexity of the utility functions. Indeed, different values of this parameter imply different levels of tolerance for risky lotteries between two outcomes. A value of $a > 1$ represents *risk-averse* parties (because it induces a concave utility function); a value of $a < 1$ represents *risk-seeking* parties (because it induces a convex utility function); and a value of $a = 1$ represents *risk-neutral* parties (because it induces a linear utility function).

It is interesting to study this parameter for several reasons. While most formal models assume that political actors are averse to risk, the opposite attitude of preferring to take on risk is also a valid assumption to make. In the real world, risk-seeking behavior often occurs in social, economic and political situations, so including it in our theories makes them more complete. Importantly, a love of risk can serve as a centrifugal force in this election, meaning that it provides some new incentives for platforms to diverge toward the extremes. To illustrate this point consider the following scenario. We should start by recalling that this model does not exhibit any asymmetric information, as the preferences of all actors are common knowledge. This includes the location of the median voter and the median party

[12] This adjustment to -1 and 1 is done for presentation purposes. It represents a slight loss of generality because it imposes that both parties have median members exactly at the same distance from the center; but we can readily prove that all the results would go through without this symmetry.

members, which are known to everyone for sure. So when does risk play a role? Suppose one of the parties has the option between nominating an extreme candidate who would tie with the extreme candidate from the other party thus making the winner uncertain, or nominating a moderate candidate who would win the election for sure. A risk-seeking party might prefer nominating the extreme candidate that induces a lottery between the two parties, rather than the moderate candidate that ensures a centrist victory. In this sense, risk-seeking behavior incentivizes extremism. It is germane to explore whether this new centrifugal force would make the median-voter theorem collapse by inducing candidates to diverge away from the center.[13]

As in many formal models, here we need to specify how indifferences are resolved. If a party knows that its two precandidates would lead to different outcomes in the election, it will always choose the one who ensures a policy closest to the party's ideal point – and it will do so with no regard to winning the election, which the party is not seeking to do. However, there are several hypothetical circumstances where a party would be indifferent between its two precandidates because they would both yield the same expected payoff. To break these indifferences, I will make the following three assumptions.

Indifference Assumptions: *Given the platform that the rival party is expected to adopt, if a party is indifferent between its two precandidates in terms of expected policy, it will chose according to the following assumptions.*

(IA1) *If both precandidates adopt the same platform making them indistinguishable, the party is forced to randomize equally between them.*

(IA2) *If both precandidates adopt different platforms, but both of them have the same probability of winning the election, the party can nominate either of them in equilibrium.*

(IA3) *If both precandidates adopt different platforms and they have different probabilities of winning the election, the party will chose the one that offers the highest probability.*

[13] The theory in Serra (2015) found that primary elections did not induce any extremism; but it only considered strictly risk-averse parties. The present chapter extends the analysis to other attitudes toward risk.

These assumptions imply that parties have a very mild preference for victory in the following sense: assumption IA3 represents a weak preference for winning the general election *per se*. It is akin to assuming that each party has a lexicographic benefit from being elected, which only plays a role when its options are indifferent in terms of policy.[14]

2.4 Candidates who value being nominated in addition to being elected

Candidates are motivated by winning electoral contests. I assume they only value the perquisites from victorious elections, such as prestige, power and material benefits. Using the political-science jargon we would say that they only care about their *ego-rents.* In particular, the candidates do not derive utility from the policy implemented. Not caring about policy gives candidates the freedom to announce any platform that best suits their goal of winning the nomination to later win the election.

In contrast to most existing models of primary elections, here I will assume that a candidate values obtaining her party's nomination *per se*, independently of winning the general election afterwards. In other words, I will assume that winning a primary election also grants some prestige, power, material benefits and other ego-rents.[15]

To be concrete, candidate i wishes to maximize the expected payoff from the primary process, labeled P, plus the expected payoff from the general election, labeled G. So she has the following utility function:

$$U_i\left(S_c, S_L, S_R\right) = E\left(P\right) + E\left(G\right)$$

[14] The first two assumptions, IA1 and IA2, were also used in Serra (2015), but the third one, IA3, was not necessary in that previous model. This mild assumption became necessary now due to the candidates' payoff from winning the nomination independently of winning the election.

[15] This represents an extension with respect to Serra (2015), which did not consider nominations to be valuable *per se.*

where P is given by

$$P = \begin{cases} p \text{ if } i \text{ wins the primary election} \\ 0 \text{ otherwise} \end{cases}$$

with $p \geq 0$

and G is given by

$$G = \begin{cases} g \text{ if } i \text{ wins the general election} \\ 0 \text{ otherwise} \end{cases}$$

with $g > 0$

So the values p and g correspond to the ego-rents received by the candidate from winning the primary and general elections, respectively.

Each candidate will choose her platform rationally, meaning she will take into account the reactions of other players. In particular, all candidates will try outguessing one another such that platform announcements form a Nash equilibrium between the four of them. They are also forward looking, meaning that they will calculate the consequences of their announcements down the line, when it is the parties' turn to nominate a candidate, and then the voters' turn to elect a party. This structure implies that candidates will try balancing their need to please their parties who have extremist ideal points, with the subsequent need, if they are nominated, to appeal to the median voter who has a centrist ideal point. They must find this balance recalling that any platform they announce in the primary will remain their platform in the general election as well.

One immediate implication is that rational candidates would only consider adopting platforms in the following intervals. Candidates r_1 and r_2 in party R will restrict themselves to the interval $[0, 1]$, while candidates l_1 and l_2 in party L will restrict themselves to the interval $[-1, 0]$.

2.5 Equilibrium concept

Our best prediction for the election result is an equilibrium of this game. We thus need to solve for all the equilibrium strategies of candidates, parties and voters. The game is solved by backward induction, and the type of equilibrium that we are looking for is *subgame perfect Nash equilibrium* (SPNE). A SPNE must induce a Nash Equilibrium (NE) in every subgame of the game, and therefore we need to find strategies S_c^*, S_L^* and S_R^* that induce an NE at every stage of the election. I will only consider pure strategies.[16]

Special focus will be placed on the location of the platforms that candidates will choose. We are particularly interested in exploring whether *complete convergence* or *large divergences* can be sustained in equilibrium. Will candidates adopt extremist platforms pandering to their parties, or will they announce centrist policies catering to the median voter? In turn, will parties nominate moderate candidates who can more easily win the election, or will they prefer partisans close to their ideal points? The following section provides answers in the context of this basic model.

3 The null effect of primaries on polarization

3.1 A median voter theorem

We can now state a new theorem about the effect of primary elections on polarization. As it turns out, in this model, even with several centrifugal forces, there is no effect at all. Complete convergence is the only equilibrium, such that all candidates adopt completely moderate platforms before the nominations take place.

Theorem *The following will hold for any values of the parties' attitude toward risk, a, and the candidates' payoffs from winning the primary election, p, and winning the general election, g. In this election, there exists a unique outcome that can be sustained in equilibrium. In this outcome, all the candidates converge to the median voter's ideal point such that $r_1 = r_2 = l_1 = l_2 = 0$. Party L randomizes between l_1 and l_2. Party R*

[16] It can be proved that allowing players to use mixed strategies would not change the results, so I ignore them in the presentation.

randomizes between r_1 and r_2. Voters randomize between party L and party R. And the policy implemented after the election is 0*, the ideal point of the median voter.*

This result is not trivial given the centrifugal forces that exist in the game. As I will illustrate below, there exist significant incentives for parties to request partisan platforms from their candidates. The theorem above shows that such centrifugal forces are more than compensated by *centripetal* forces incentivizing those same parties to converge to the center. In effect, this theorem is a generalization to primary elections of the classic median voter theorem. The result makes the same predictions as the theorem in Serra (2015), but now with more general assumptions, so it can be considered a "robust" result.

The formal proof of this result comes in the appendix. It is fairly long as it must study three separate cases corresponding to risk-averse, risk-seeking and risk-neutral parties. So to gain insight into this type of elections, I give a shorter and more intuitive explanation in the following lines.

3.2 Election dynamics

Insight can come from analyzing the different forces in this election. In particular, it is worth understanding all the options that players in this game had, and why none of these options was an equilibrium save for the ones described in the theorem. We need to analyze all the possible combinations of strategies to discard those not forming an equilibrium, namely those where at least one player could benefit from unilaterally changing her decision keeping the decision of the other players fixed. In particular, we must analyze all the possible configurations of four platforms, two in the left-wing party and two in the right-wing party, to see whether rational candidates could conceivably announce them. The following six configurations are representative of the typical dynamics in this election.[17]

- **Example 1**: $0 \leq r_1 < r_2 < -l_1 < -l_2 \leq 1$
  - Profitable deviation: $r_1 \to r_2 + \varepsilon$

[17] This is not an exhaustive list of all the possible configurations. In this section, I only analyze the cases that build an interesting intuition. The proof in the appendix gives the exhaustive list of configurations and determines whether each of them is an equilibrium or not.

– Is it an equilibrium? No

In this configuration, all candidates have announced platforms with different levels of extremism. Both left-wing candidates are more extreme than the right-wing candidates. If candidates were considering this configuration, there would be a strong centrifugal force in the election incentivizing some of the candidates to move even further away from the median voter. To see this, consider the incentives of candidate r_1. Should this become the actual configuration of platforms, party R would be sure to win the election with either of its candidates r_1 or r_2. It could thus safely nominate the candidate closest to its ideal point, r_2, and still win the election. In this case, the centrifugal incentives would dominate inside party R such that the most moderate candidate r_1 would lose the nomination in favor of the relatively more partisan candidate r_2. Being rational and forward looking, r_1 would want to avoid this outcome by moving toward its party's ideal point in order to steal the nomination from r_2. All things equal, r_1 would benefit from adopting a platform $r_2 + \varepsilon$ where ε is a small positive number, such that her platform is larger than r_2 to be more appealing to R, while still being more moderate than l_1 in order to win the election. Given that r_1 has this profitable unilateral deviation, this configuration cannot be an equilibrium.[18]

- **Example 2**: $0 = r_1 < r_2 = -l_1 = -l_2 \leq 1$
  - Profitable deviation: If parties are risk-seeking, then $r_1 \rightarrow r_2$
  - Is it an equilibrium? No

In this situation, the parties' attitude toward risk will play a crucial role. Let us assume throughout the example that $a < 1$ such that parties are risk-seeking. There is nothing to say about party L's decision given that its two precandidates are indistinguishable, forcing it to randomize between l_1 and l_2 (as postulated by the assumption IA1 above.) Party R's decision is the interesting one as it presents a dilemma. On one hand, by nominating r_1 it would secure victory for sure with a policy of zero. On the other hand, by nominating r_2 it would induce a lottery where both parties would tie and hence each one would win the

[18]This example corresponds to Configurations 15 and 16 in Table 1 in the Appendix.

election with equal probability. Given that both parties would have platforms exactly on opposite sides of the median voter, the expected policy from this lottery would be zero. Here is where risk-seeking behavior will create a centrifugal force. If party R is a risk-seeker, it will prefer the lottery instead of the sure outcome, and hence it will nominate r_2 instead of r_1. Anticipating this outcome, it is clear that r_1 would not be satisfied with her choice because she would lose the nomination. She would prefer to deviate away from the center all the way to r_2's platform, in order to tie for the nomination and obtain a strictly positive probability winning the election. Given that at least one candidate wishes to change her location, this is not an equilibrium.[19] Incidentally, it should be noted that party L has two different choices under this configuration, but both of them would lead to the same outcome, i.e., losing the election to R. According to the assumption IA2, then either l_1 or l_2 could be nominated by L.

- **Example 3**: $0 < r_1 < -l_1 < r_2 < -l_2 \leq 1$
  - Profitable deviation: $l_2 \to -r_1 + \varepsilon$
  - Is it an equilibrium? No

This configuration would create *centripetal* forces in the election, meaning that candidates would have an incentive to become more moderate than they were planning. To see this, consider how nominations would play out in parties L and R. In principle, party L would find candidate l_2 most attractive as she is located closest the its ideal point. This is the candidate that party L would nominate if it was sincere instead of strategic. However, we postulated that both parties are rational hence anticipating each other's strategies. If party L was planning to nominate l_2, R's best response would be to nominate r_2, but then L's best response would be to nominate l_1, in which case R's best response would be to nominate r_1. Hence both parties will "race toward the center". With rational parties, the two moderate precandidates will be nominated at the expense of the two partisan ones. What incentives does this create for candidate l_2? Given that she would lose the nomination under this configuration of announcements by the other candidates, she would prefer to adopt a

[19] This example corresponds to Configuration 7 in Table 1 in the Appendix.

drastically more moderate platform, such as $-r_1 + \varepsilon$ where ε is a small positive number. If she did so, competition with R would force L to nominate the new l_2 in order to win the election. This incentive for the most partisan candidate to become the most moderate one illustrates the strong centripetal force in this election, and discards this configuration as a possible equilibrium.[20]

- **Example 4**: $0 < r_1 = r_2 = -l_1 = -l_2 = 1$
  - Profitable deviation: $r_1 \to r_1 - \varepsilon$
  - Is it an equilibrium? No

In this configuration, all candidates have adopted the ideal points of their respective parties, that is, they have located at $l_1 = l_2 = -1$ and $r_1 = r_2 = 1$, which we assumed to be the ideal points of the median party members in L and R. One possibility for this configuration is that candidates assumed –mistakenly– that their parties were sincere instead of strategic. In this case, parties would face identical precandidates such that L would not be able to distinguish between l_1 and l_2, and R would not be able to distinguish between r_1 and r_2. Parties would not really have a substantive choice, so they would simply randomize between their precandidates giving them an equal chance of being nominated. Following the nominations, both parties will have candidates whose platforms are exactly equidistant from the median voter, hence tying in the election with an equal chance of winning. Candidates would have equal expected payoffs: if no candidate deviates from this agreement, each one can expect a probability of $\frac{1}{2}$ of being nominated, and a probability of $\frac{1}{2}$ of winning the election conditional on being nominated. However, the candidates cannot sustain this configuration in equilibrium, as each of them would benefit from deviating unilaterally to a slightly more moderate platform. For example, if r_1, moved infinitesimally toward the center, she would give party R the opportunity to nominate her to subsequently win the election for sure with a right-wing platform, instead of tying with L's left-wing platform. Hence this profile is not a NE.[21]

[20] This example corresponds to Configuration 24 in Table 1 in the Appendix.
[21] This example corresponds to Configuration 2 in Table 1 in the Appendix.

- **Example 5**: $0 = r_1 = r_2 = l_1 < -l_2 \leq 1$
  - Profitable deviation: $l_2 \to 0$
  - Is it an equilibrium? No

Here every candidate has decided to converge completely to the center except for l_2 who has decided to remain more partisan. Irrespective of party L's nominee, party R will have to nominate a completely centrist candidate. So candidate l_2 is sure to lose the general election with her partisan platform, thus forgoing any chance of earning the payoff g. Can we imagine a justification for l_2 to adopt this non-centrist platform? Perhaps she hopes to win the nomination over l_1, thus receiving the payoff p for sure, even if she then loses the subsequent election. However, this calculation is misguided for the following reason. In this configuration, party L has the choice between two precandidates with different platforms. Yet the outcome would be the same irrespective of whom it nominates: the policy implemented will be zero either way. So L is indifferent in terms of policy between its two precandidates, and our indifference assumptions kick in. I had assumed in IA3 that in cases such as this one the party will chose the candidate that offers the highest chance of winning the election. This corresponds to l_1 who would have a probability of $\frac{1}{2}$ of beating R's candidates, rather than l_2 whose probability is zero. Therefore, under this configuration, L will nominate l_1. Given that l_2 would currently receive a zero payoff, she would benefit from deviating all the way to the center of the spectrum to achieve strictly positive probabilities of being nominated and winning the election. Hence this is not an equilibrium.[22]

- **Example 6**: $0 = r_1 = r_2 = l_1 = l_2$
  - Profitable deviation: None
  - Is it an equilibrium? Yes

In this configuration, all the candidates have converged fully to the median voter. Neither party has a choice for the nomination given that all precandidates are indistinguishable.

[22] This example corresponds to Configuration 3 in Table 1 in the Appendix.

Party L has no choice but to randomize between l_1 and l_2, while party R has no choice but to randomize between r_1 and r_2. Following the primaries, the median voter will face parties with identical platforms, and will hence randomize between the two. The policy implemented after the election will be 0, the ideal point of M. If no candidate deviates from this configuration, each candidate has a probability of $\frac{1}{2}$ of being nominated, and a probability of $\frac{1}{2}$ of winning the election conditional on being nominated. However, if any candidate, say l_2, deviated unilaterally to become slightly more extremist, parties would face the situation described in Example 5. We know from the analysis above that l_2 would not be nominated and hence would not win the election. Given that she would lose the chance of earning the payoff p and then the payoff g, this deviation is not profitable. Since this is true of all other candidates as well, none of them have a profitable unilateral deviation. So this represents an equilibrium – the only one in this election.[23]

These heuristic examples should convey intuitively why the theorem holds. (The complete formal proof comes in the appendix.) Now we are in a better position to discuss the implications of these results.

4 Discussion: Finding out if primaries really create polarization

Adopting primary elections is thought to bring benefits to parties and to the party system as a whole such as, notably, democratizing the selection of candidates which can otherwise be a quite undemocratic process. In spite of these benefits, there have been worries about some social costs that primary elections might carry. One such cost is alleged to be a larger polarization between the policies advocated by candidates from different parties. In the American-politics literature, it is often conjectured that CSMs with more exclusive selectorates will nominate candidates with more extreme platforms. For example, closed primaries, where only party members can vote, are presumed to elect more extremist nominees than open primaries, where non-registered citizens can vote. One alleged reason is that

[23]This example corresponds to Configuration 1 in Table 1 in the Appendix.

party members are typically composed of passionate activists rather than dispassionate moderates. A first set of statistical studies seemed to confirm this conjecture,[24] which spawned a theoretical literature claiming that primary elections cause candidates to diverge.[25]

However, the early empirical studies and the subsequent formal models are contradicted by the newer statistical research. A number of studies have been reporting a null finding in their correlations between closed primaries and candidate divergence.[26] How can we make sense of these contradictory findings? One way is to use formal theory to try shedding light on this complex issue. My interpretation of the existing findings is that primary elections might have a contingent effect, causing polarization only under certain conditions. If so, it would be useful to have formal models predicting full convergence, meaning that primary candidates move to the center of the political spectrum. Theories of this kind could then be contrasted with theories making the opposite prediction to better understand the respective assumptions that led to their contradiction. This comparison would enable a better understating of candidate selection, at least theoretically, which in turn could hopefully motivate further empirical studies.

This was the objective of my model in this chapter. In an attempt to be straightforward, I endeavored to add only essential features of primary elections to the well-established spatial-voting model of Downs (1957). Before the standard election between two parties, I added a previous stage where each party holds a competitive nomination between two precandidates. The model included at least five features that can be thought of creating "centrifugal" forces: (1) parties with ideal points on opposite sides of the median voter; (2) parties that are policy-motivated instead of office-motivated; (3) primary platforms that are sticky throughout the election; (4) an independent payoff to candidates from winning the primary; and (5) risk-seeking parties that prefer a lottery between two extremist candidates to winning the election with a moderate candidate.

Given these centrifugal forces, it is surprising that we did not find any polarization whatsoever. The result in this model is a median-voter theorem with primary elections, whereby

[24] Such as Gerber and Morton (1998).

[25] Such as those mentioned in footnote 4. I make a more extensive literature review in Serra (2018).

[26] Such as Hirano, Snyder, Ansolabehere and Hansen (2010); Peress (2013); and McGhee, Masket, Shor, Rogers and McCarty (2014).

all the candidates during the nomination process announce completely centrist platforms. What can account for this counter-intuitive result? It must be that some centripetal forces have more-than-compensated the centrifugal ones. One of the most important forces illustrated by this model is the rationality of party members. My hypothesis about real-life primary elections is that strategic voting represents a powerful force driving parties towards the center. According to this view, it should not matter whether primary voters are super extremist: if these primary voters are strategic, my model predicts that precandidates within the party will converge to the median voter. The same result was found in Serra (2015), namely that fully strategic parties should locate at the center in spite of holding primaries. While that previous research included the centrifugal forces (1), (2) and (3) mentioned above, this chapter is original in adding (4) and (5). Given that I find the same result, namely the complete convergence to the center of all candidates, the model in this chapter can be considered a robustness check of the model in Serra (2015).

These results, however, remain silent about boundedly-rational primary voters. If they voted sincerely, maybe some polarization would occur. In fact, I am finding theoretical support for this hypothesis in parallel research: in Serra (2017), I find that sincere voting in primary elections leads to more divergence than strategic voting. A final implication of this research can hopefully be conveyed: according to these findings, empirical research should pay more attention to a neglected variable, namely the degree of rationality of party members whereby they will vote strategically or sincerely in primaries. If this variable was contemplated more often, I believe we could make further progress in this debate.

A Appendix: Proof of the theorem

A.1 General considerations

Without loss of generality, the configurations in Table 1, along with their symmetric counterparts, are an exhaustive list of all the possible configurations of platforms that candidates may adopt. All cases are mutually exclusive. We assume that $l_1, l_2 \in [-1, 0]$ and $r_1, r_2 \in [0, 1]$.

Table 1	
Configuration 1	$0 = r_1 = r_2 = -l_1 = -l_2$
Configuration 2	$0 < r_1 = r_2 = -l_1 = -l_2$
Configuration 3	$0 = r_1 = r_2 = -l_1 < -l_2$
Configuration 4	$0 < r_1 = r_2 = -l_1 < -l_2$
Configuration 5	$0 = r_1 = r_2 < -l_1 = -l_2$
Configuration 6	$0 < r_1 = r_2 < -l_1 = -l_2$
Configuration 7	$0 = r_1 < r_2 = -l_1 = -l_2$
Configuration 8	$0 < r_1 < r_2 = -l_1 = -l_2$
Configuration 9	$0 = r_1 = r_2 < -l_1 < -l_2$
Configuration 10	$0 < r_1 = r_2 < -l_1 < -l_2$

Table 1 (continued)	
Configuration 11	$0 = r_1 < r_2 = -l_1 < -l_2$
Configuration 12	$0 < r_1 < r_2 = -l_1 < -l_2$
Configuration 13	$0 = r_1 < r_2 < -l_1 = -l_2$
Configuration 14	$0 < r_1 < r_2 < -l_1 = -l_2$
Configuration 15	$0 = r_1 < r_2 < -l_1 < -l_2$
Configuration 16	$0 < r_1 < r_2 < -l_1 < -l_2$
Configuration 17	$0 = r_1 = -l_1 < r_2 = -l_2$
Configuration 18	$0 < r_1 = -l_1 < r_2 = -l_2$
Configuration 19	$0 = r_1 = -l_1 < r_2 < -l_2$
Configuration 20	$0 < r_1 = -l_1 < r_2 < -l_2$

Table 1 (continued)	
Configuration 21	$0 = r_1 < -l_1 < r_2 = -l_2$
Configuration 22	$0 < r_1 < -l_1 < r_2 = -l_2$
Configuration 23	$0 = r_1 < -l_1 < r_2 < -l_2$
Configuration 24	$0 < r_1 < -l_1 < r_2 < -l_2$
Configuration 25	$0 = r_1 < -l_1 = -l_2 < r_2$
Configuration 26	$0 < r_1 < -l_1 = -l_2 < r_2$
Configuration 27	$0 = r_1 < -l_1 < -l_2 < r_2$
Configuration 28	$0 < r_1 < -l_1 < -l_2 < r_2$

With this list in mind, I proceed to prove the theorem in this chapter. The proof needs to be separated in three cases corresponding to risk-averse parties ($a > 1$), risk-seeking parties ($a < 1$), and risk-neutral parties ($a = 1$).

A.2 Proof with risk-averse parties ($a > 1$)

The game must be solved by backward induction. The procedure will be the following: we start by solving the game at its last stage –the general election– and we find the median voter's strategy profile that forms a NE in every situation in which she might be called upon to act. Given this strategy by the median voter, we consider the reduced game at the second stage –the nominations by each party– and we find the strategies S_L^* and S_R^* that form a NE for the parties in every possible subgame in which they might be called upon to act. Finally, for each S_L^* and S_R^*, we consider the reduced game at its first stage –the platform adoption– and we find all the strategies S_c^* that form a NE for the candidates. At this stage (the platform adoption), we know that a NE of the reduced game will be a SPNE of the game as a whole. This subsection will carry out this procedure for the case $a > 1$. The following two sections will carry out the same procedure for the cases $a = 1$ and $a < 1$.

- **Third stage**

First we prove that sincere voting is a weakly dominant strategy for voters. When casting her ballot, a voter is either pivotal or not. If she is pivotal, then voting other than sincerely will make her worse off (or no better off if she is indifferent between both parties). If her vote is not pivotal then any strategy leads to the same outcome. Therefore, sincere voting is never worse and sometimes better than not voting sincerely. Thus, sincere voting weakly dominates every other strategy for voters. If we assume that a voters will never choose a weakly dominated strategy, they will vote sincerely. Given that the preferences of voters are symmetric and single peaked, and that we assumed the existence of a median voter, the electorate will behave according to the preferences of this median voter. There are two possible subgames: either $r_i = -l_j$ or $r_i \neq -l_j$. In the latter case, the candidate closer to zero will win the election. In the former case, there is a tie between the candidates, and the median voter will decide by flipping a coin.

- **Second stage**

Without loss of generality, the configurations in Table 2, along with their symmetric counterparts, are an exhaustive list of all the possible subgames that parties may face, along with their corresponding Nash equilibria. I only list the equilibria in pure strategies. (Analyzing the possible mixed-strategy equilibria would not change the results, so I ignore them in this proof.) In this list, the pair of strategies (l_i, r_j) refers to the decision of party L to nominate l_i in conjunction with the decision of party R to nominate r_j. The strategy labeled "rand" is used when a party is forced to randomize equally between its two candidates because they are indistinguishable.

It will be important to keep in mind the indifference assumptions described in the text, which said the following. Given the platform that the rival party is expected to adopt, if a party is indifferent between its two precandidates in terms of expected policy, it will chose the one that offers the highest chance of winning the election (IA3). If both offer the same chance of winning the election and they both have distinct platforms, then either can be chosen in equilibrium (IA2). And if they both adopted the same platform making them indistinguishable, the party is forced to randomize between them (IA1). The third column lists all the NE in each subgame without applying this indifference assumption. The fourth column lists all the NE after eliminating those not conforming to the assumption IA3.

Table 2 – Equilibria between parties – risk-averse case			
		NE without IA3	NE with IA3
Subg. 1	$0 = r_1 = r_2 = -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 2	$0 < r_1 = r_2 = -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 3	$0 = r_1 = r_2 = -l_1 < -l_2$	(l_1, rand) and (l_2, rand)	(l_1, rand)
Subg. 4	$0 < r_1 = r_2 = -l_1 < -l_2$	(l_1, rand)	(l_1, rand)
Subg. 5	$0 = r_1 = r_2 < -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 6	$0 < r_1 = r_2 < -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 7	$0 = r_1 < r_2 = -l_1 = -l_2$	(rand, r_1)	(rand, r_1)
Subg. 8	$0 < r_1 < r_2 = -l_1 = -l_2$	(rand, r_1)	(rand, r_1)
Subg. 9	$0 = r_1 = r_2 < -l_1 < -l_2$	(l_1, rand) and (l_2, rand)	(l_1, rand) and (l_2, rand)
Subg. 10	$0 < r_1 = r_2 < -l_1 < -l_2$	(l_1, rand) and (l_2, rand)	(l_1, rand) and (l_2, rand)

Table 2 – Equilibria between parties – risk-averse case (continued)			
		NE without IA3	NE with IA3
Subg. 11	$0 = r_1 < r_2 = -l_1 < -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 12	$0 < r_1 < r_2 = -l_1 < -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 13	$0 = r_1 < r_2 < -l_1 = -l_2$	(rand, r_2)	(rand, r_2)
Subg. 14	$0 < r_1 < r_2 < -l_1 = -l_2$	(rand, r_2)	(rand, r_2)
Subg. 15	$0 = r_1 < r_2 < -l_1 < -l_2$	(l_1, r_2) and (l_2, r_2)	(l_1, r_2) and (l_2, r_2)
Subg. 16	$0 < r_1 < r_2 < -l_1 < -l_2$	(l_1, r_2) and (l_2, r_2)	(l_1, r_2) and (l_2, r_2)
Subg. 17	$0 = r_1 = -l_1 < r_2 = -l_2$	(l_2, r_1) and (l_1, r_2) and (l_1, r_1)	(l_1, r_1)
Subg. 18	$0 < r_1 = -l_1 < r_2 = -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 19	$0 = r_1 = -l_1 < r_2 < -l_2$	(l_1, r_1) and (l_1, r_2)	(l_1, r_1)
Subg. 20	$0 < r_1 = -l_1 < r_2 < -l_2$	(l_1, r_1)	(l_1, r_1)

Table 2 – Equilibria between parties – risk-averse case (continued)			
		NE without IA3	NE with IA3
Subg. 21	$0 = r_1 < -l_1 < r_2 = -l_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)
Subg. 22	$0 < r_1 < -l_1 < r_2 = -l_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)
Subg. 23	$0 = r_1 < -l_1 < r_2 < -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 24	$0 < r_1 < -l_1 < r_2 < -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 25	$0 = r_1 < -l_1 = -l_2 < r_2$	(rand, r_1)	(rand, r_1)
Subg. 26	$0 < r_1 < -l_1 = -l_2 < r_2$	(rand, r_1)	(rand, r_1)
Subg. 27	$0 = r_1 < -l_1 < -l_2 < r_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)
Subg. 28	$0 < r_1 < -l_1 < -l_2 < r_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)

To be part of a SPNE, any strategy profile S_L^* and S_R^* must induce these NE in the corresponding subgames. Note that several subgames admit two equilibria in pure strategies.

Out of those, subgames 3, 17 and 19 are reduced to having a unique equilibrium upon applying the indifference assumption IA3; and the rest remain with two NE even after applying the indifference assumptions.

To illustrate how this table was derived, I will solve Subgame 3. Party R does not have a real choice since both of its candidates have adopted indistinguishable platforms. According to assumption IA1, its unique available strategy is to randomize between r_1 and r_2. On the other hand, party L has a choice between $l_1 = 0$ and $l_2 > 0$. If L nominates l_1 it will tie with R and the policy implemented will be 0 for sure. If L nominates l_2 it will lose against R and the policy implemented will be 0 for sure as well. Hence, both nominations lead to the same policy outcome and give L the same utility. Therefore, in terms of policy, L is indifferent between l_1 and l_2, which leads to two possible Nash equilibria in pure strategies: (l_1, rand) and (l_2, rand). However, the second one will be eliminated by our indifference assumption IA3. According to this assumption, L has a lexicographic preference for victory that only plays a role when l_1 and l_2 would yield the same payoff in terms of policy. By nominating l_1 the party's probability of wining would be one half, whereas by nominating l_2 it would be zero. Consequently, l_1 will be nominated instead of l_2, and the only NE that survives is (l_1, rand).

It is also illustrative to solve Subgame 10. Party R does not have a choice because its two precandidates chose identical platforms. Party L can choose between two different candidates but, interestingly, they would both lead to the same outcome, namely, they would both lose against the candidate from the rival party. That's because either r_1 or r_2 are strictly closer to the median voter than both l_1 and l_2. According to assumption IA2, Party L can nominate either of its precandidates in equilibrium, which explains why this profile has two Nash equilibria in pure strategies: (l_1, rand) and (l_2, rand).

Analysis of the other 26 subgames follows similar steps.

- **First stage**

Without loss of generality, the profiles in Table 3, along with their symmetric counterparts, are an exhaustive list of all the possible profiles of platforms that candidates may adopt, along with a profitable deviation, if any. Below, ε is a strictly positive number that is infinitesimally small.

Table 3 – Equilibria between candidates – risk-averse case			
		Profitable deviation	Is it a Nash equilibrium?
Profile 1	$0 = r_1 = r_2 = -l_1 = -l_2$	None	Yes
Profile 2	$0 < r_1 = r_2 = -l_1 = -l_2$	$r_1 \to r_1 - \varepsilon$	No
Profile 3	$0 = r_1 = r_2 = -l_1 < -l_2$	$l_2 \to 0$	No
Profile 4	$0 < r_1 = r_2 = -l_1 < -l_2$	$l_2 \to l_1 + \varepsilon$	No
Profile 5	$0 = r_1 = r_2 < -l_1 = -l_2$	$r_2 \to r_2 + \varepsilon$	No
Profile 6	$0 < r_1 = r_2 < -l_1 = -l_2$	$r_2 \to r_2 + \varepsilon$	No
Profile 7	$0 = r_1 < r_2 = -l_1 = -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 8	$0 < r_1 < r_2 = -l_1 = -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 9	$0 = r_1 = r_2 < -l_1 < -l_2$	$r_2 \to r_2 + \varepsilon$	No
Profile 10	$0 < r_1 = r_2 < -l_1 < -l_2$	$r_2 \to r_2 + \varepsilon$	No

Table 3 – Equilibria between candidates – risk-averse case (continued)			
		Profitable deviation	Is it a Nash equilibrium?
Profile 11	$0 = r_1 < r_2 = -l_1 < -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 12	$0 < r_1 < r_2 = -l_1 < -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 13	$0 = r_1 < r_2 < -l_1 = -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 14	$0 < r_1 < r_2 < -l_1 = -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 15	$0 = r_1 < r_2 < -l_1 < -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 16	$0 < r_1 < r_2 < -l_1 < -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 17	$0 = r_1 = -l_1 < r_2 = -l_2$	$r_2 \to 0$	No
Profile 18	$0 < r_1 = -l_1 < r_2 = -l_2$	$r_2 \to r_1 - \varepsilon$	No
Profile 19	$0 = r_1 = -l_1 < r_2 < -l_2$	$l_2 \to 0$	No
Profile 20	$0 < r_1 = -l_1 < r_2 < -l_2$	$r_2 \to r_1 - \varepsilon$	No

Table 3 – Equilibria between candidates – risk-averse case (continued)			
		Profitable deviation	Is it a Nash equilibrium?
Profile 21	$0 = r_1 < -l_1 < r_2 = -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 22	$0 < r_1 < -l_1 < r_2 = -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 23	$0 = r_1 < -l_1 < r_2 < -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 24	$0 < r_1 < -l_1 < r_2 < -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 25	$0 = r_1 < -l_1 = -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 26	$0 < r_1 < -l_1 = -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 27	$0 = r_1 < -l_1 < -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 28	$0 < r_1 < -l_1 < -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No

I will prove why Profile 1 is an equilibrium for candidates and therefore a solution to this game. Suppose none of the candidates deviated from it. Then parties would face Subgame

1, and we can see from Table 2 that each party randomizes between their candidates. So each candidate has a probability of $\frac{1}{2}$ to be nominated and a probability of $\frac{1}{2}$ of to win the election conditional on being nominated. Their expected payoff is thus $\frac{p}{2} + \frac{g}{4}$. Suppose, on the other hand, that one of the candidates deviated *unilaterally*. Then parties would face Subgame 3 or its symmetrical counterpart. The candidate that deviated might have been hoping to win the nomination for sure to receive a payoff p even if she later loses the election. However, this outcome would not materialize. As I analyzed above, the unique NE that survives our the indifference assumptions in Subgame 3 is the one where the centrist candidates are nominated in detriment of the extremist candidates. Hence the candidate who deviated would lose the nomination and would obtain a zero payoff. Such a deviation is therefore not profitable, and the configuration is a NE.

Now I will prove that Profile 10 is not an equilibrium. In particular, I will show that a centrifugal force exists in this situation whereby candidate r_2 would prefer to become more extremist. As I analyzed above, if this profile is adopted then both candidates within party R would be tied to get the nomination, and whoever gets the nomination is sure to win the election. However, if r_2 moved to the right an infinitesimal amount, candidates would fall in Profile 16. According to Table 2 (and also Example 1 in the main text), in this profile, r_2 would secure the nomination for sure and would then win the election. This is an improvement for the candidate, so Profile 10 cannot be a NE.

Analysis of the remaining subgames follows similar steps: it can be proved that none of them is a Nash equilibrium (see the profitable deviations in each case). Thus Profile 1 is the unique NE of the reduced game, and it is the unique strategy profile of candidates that can be part of a SPNE. Therefore in any strategy profile S_c^*, S_L^* and S_R^* that forms a SPNE, the outcome will be the same: candidates adopt the platforms in Profile 1, which are $0 = r_1 = r_2 = -l_1 = -l_2$. This is exactly what the theorem says.

A.3 Proof with risk-seeking parties ($a < 1$)

Once again, we solve the game by backward induction.

- **Third stage**

Sincere voting is a weakly dominant strategy for the voters. The proof is the same as for risk-averse parties.

- **Second stage**

In Table 4, we list again all the possible subgames that parties may face, along with their corresponding NE. Note the use of *and* when several equilibria are possible, and *or* when only one out of two different equilibria is possible:

Table 4 – Equilibria between parties – risk-seeking case

		NE without IA3	NE with IA3
Subg. 1	$0 = r_1 = r_2 = -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 2	$0 < r_1 = r_2 = -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 3	$0 = r_1 = r_2 = -l_1 < -l_2$	(l_1, rand) and (l_2, rand)	(l_1, rand)
Subg. 4	$0 < r_1 = r_2 = -l_1 < -l_2$	(l_1, rand)	(l_1, rand)
Subg. 5	$0 = r_1 = r_2 < -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 6	$0 < r_1 = r_2 < -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 7	$0 = r_1 < r_2 = -l_1 = -l_2$	(rand, r_2)	(rand, r_2)
Subg. 8	$0 < r_1 < r_2 = -l_1 = -l_2$	(rand, r_1) or (rand, r_2)	(rand, r_1) or (rand, r_2)
Subg. 9	$0 = r_1 = r_2 < -l_1 < -l_2$	(l_1, rand) and (l_2, rand)	(l_1, rand) and (l_2, rand)
Subg. 10	$0 < r_1 = r_2 < -l_1 < -l_2$	(l_1, rand) and (l_2, rand)	(l_1, rand) and (l_2, rand)

Table 4 – Equilibria between parties – risk-seeking case (continued)			
		NE without IA3	NE with IA3
Subg. 11	$0 = r_1 < r_2 = -l_1 < -l_2$	(l_1, r_2)	(l_1, r_2)
Subg. 12	$0 < r_1 < r_2 = -l_1 < -l_2$	(l_1, r_1) or (l_1, r_2)	(l_1, r_1) or (l_1, r_2)
Subg. 13	$0 = r_1 < r_2 < -l_1 = -l_2$	(rand, r_2)	(rand, r_2)
Subg. 14	$0 < r_1 < r_2 < -l_1 = -l_2$	(rand, r_2)	(rand, r_2)
Subg. 15	$0 = r_1 < r_2 < -l_1 < -l_2$	(l_1, r_2) and (l_2, r_2)	(l_1, r_2) and (l_2, r_2)
Subg. 16	$0 < r_1 < r_2 < -l_1 < -l_2$	(l_1, r_2) and (l_2, r_2)	(l_1, r_2) and (l_2, r_2)
Subg. 17	$0 = r_1 = -l_1 < r_2 = -l_2$	(l_1, r_1) and (l_2, r_2)	(l_1, r_1) and (l_2, r_2)
Subg. 18	$0 < r_1 = -l_1 < r_2 = -l_2$	(l_1, r_1) and possibly (l_2, r_2)	(l_1, r_1) and possibly (l_2, r_2)
Subg. 19	$0 = r_1 = -l_1 < r_2 < -l_2$	(l_1, r_1) and (l_1, r_2)	(l_1, r_1)
Subg. 20	$0 < r_1 = -l_1 < r_2 < -l_2$	(l_1, r_1)	(l_1, r_1)

Table 4 – Equilibria between parties – risk-seeking case (continued)			
		NE without IA3	NE with IA3
Subg. 21	$0 = r_1 < -l_1 < r_2 = -l_2$	(l_1, r_1) and possibly (l_2, r_2)	(l_1, r_1) and possibly (l_2, r_2)
Subg. 22	$0 < r_1 < -l_1 < r_2 = -l_2$	(l_1, r_1) and possibly (l_2, r_2)	(l_1, r_1) and possibly (l_2, r_2)
Subg. 23	$0 = r_1 < -l_1 < r_2 < -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 24	$0 < r_1 < -l_1 < r_2 < -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 25	$0 = r_1 < -l_1 = -l_2 < r_2$	(rand, r_1)	(rand, r_1)
Subg. 26	$0 < r_1 < -l_1 = -l_2 < r_2$	(rand, r_1)	(rand, r_1)
Subg. 27	$0 = r_1 < -l_1 < -l_2 < r_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)
Subg. 28	$0 < r_1 < -l_1 < -l_2 < r_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)

As we can see by comparing Table 4 for risk-seeking parties with Table 2 for risk-averse parties, all the subgames are solved the same way except for Subgames 7, 8, 11, 12, 17, 18, 21 and 22.

As an illustration, I will derive the Nash equilibria in Subgame 8. Party L does not have a real choice for nomination since both l_1 and l_2 have adopted indistinguishable platforms. Thus its only choice is to randomize between its two candidates. On the other hand, party R may nominate r_1 and win the election for sure, or nominate r_2 and tie with party L thus inducing a lottery between both parties. The preferred choice for R depends on the exact position of r_1. To be concrete, we need to think of a value called the *certainty equivalent*. This corresponds to the policy location that would give R the exact same payoff as a random draw between r_2 and $-r_2$. Since parties are risk-seekers, we know that the *certainty equivalent* for R is to the right of zero. If r_1 is to the left of that certainty equivalent, then R will prefer to take on risk by nominating r_2; and the unique NE will be (rand, r_2). If r_1 is to the right of that certainty equivalent, then R will prefer to win for sure by nominating r_1; and the unique NE will be (rand, r_1). If r_1 is exactly at that certainty equivalent, then R will be indifferent between r_1 and r_2; and the assumption IA3 dictates that (rand, r_1) should be the NE.

Subgame 17 is also worth studying explicitly. Suppose L nominates l_2. Then if R nominates r_1 it will win the election for sure with a policy of zero for certain. But if it nominates r_2 it will tie with L thus inducing a lottery between two divergent policies with an average location of zero. Given that parties are risk-seeking, R prefers nominating the extremist but uncertain candidate r_2 instead of the centrist but riskless candidate r_1. The same logic applies to L so (l_2, r_2) is a NE. This result illustrates how a love of risk can act as a centrifugal force. This is not the only equilibrium, however. Suppose L nominates l_1. Then R is indifferent in terms of policy between its two precandidates, because they would both lead for sure to the same policy of zero. However, r_1 would have a $\frac{1}{2}$ probability of winning the election whereas r_2 would have none; so the assumption IA3 dictates that r_1 should be chosen. The same logic applies to L so (l_1, r_1) is a NE. Which of the two equilibria will occur? In this context it is impossible to know, so both Nash equilibria are valid predictions.

The derivation of the equilibria in the remaining 26 subgames follows the same logic.

- **First stage**

In Table 5, we list again all the possible profiles of platforms that candidates may adopt, along with a profitable deviation, if any.

Table 5 – Equilibria between candidates – risk-seeking case			
		Profitable deviation	Is it a Nash equilibrium?
Profile 1	$0 = r_1 = r_2 = -l_1 = -l_2$	None	Yes
Profile 2	$0 < r_1 = r_2 = -l_1 = -l_2$	$r_1 \to r_1 - \varepsilon$	No
Profile 3	$0 = r_1 = r_2 = -l_1 < -l_2$	$l_2 \to 0$	No
Profile 4	$0 < r_1 = r_2 = -l_1 < -l_2$	$l_2 \to l_1 + \varepsilon$	No
Profile 5	$0 = r_1 = r_2 < -l_1 = -l_2$	$r_2 \to r_2 + \varepsilon$	No
Profile 6	$0 < r_1 = r_2 < -l_1 = -l_2$	$r_2 \to r_2 + \varepsilon$	No
Profile 7	$0 = r_1 < r_2 = -l_1 = -l_2$	$r_1 \to r_2$	No
Profile 8	$0 < r_1 < r_2 = -l_1 = -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 9	$0 = r_1 = r_2 < -l_1 < -l_2$	$r_2 \to r_2 + \varepsilon$	No
Profile 10	$0 < r_1 = r_2 < -l_1 < -l_2$	$r_2 \to r_2 + \varepsilon$	No

Table 5 – Equilibria between candidates – risk-seeking case (continued)			
		Profitable deviation	Is it a Nash equilibrium?
Profile 11	$0 = r_1 < r_2 = -l_1 < -l_2$	$r_1 \to r_2$	No
Profile 12	$0 < r_1 < r_2 = -l_1 < -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 13	$0 = r_1 < r_2 < -l_1 = -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 14	$0 < r_1 < r_2 < -l_1 = -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 15	$0 = r_1 < r_2 < -l_1 < -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 16	$0 < r_1 < r_2 < -l_1 < -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 17	$0 = r_1 = -l_1 < r_2 = -l_2$	If the NE is (l_1, r_1) then $r_2 \to 0$ If the NE is (l_2, r_2) then $l_1 \to l_2$	No
Profile 18	$0 < r_1 = -l_1 < r_2 = -l_2$	$r_2 \to r_1 - \varepsilon$	No
Profile 19	$0 = r_1 = -l_1 < r_2 < -l_2$	$l_2 \to 0$	No
Profile 20	$0 < r_1 = -l_1 < r_2 < -l_2$	$r_2 \to r_1 - \varepsilon$	No

Table 5 – Equilibria between candidates – risk-seeking case (continued)			
		Profitable deviation	Is it a Nash equilibrium?
Profile 21	$0 = r_1 < -l_1 < r_2 = -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 22	$0 < r_1 < -l_1 < r_2 = -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 23	$0 = r_1 < -l_1 < r_2 < -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 24	$0 < r_1 < -l_1 < r_2 < -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 25	$0 = r_1 < -l_1 = -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 26	$0 < r_1 < -l_1 = -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 27	$0 = r_1 < -l_1 < -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 28	$0 < r_1 < -l_1 < -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No

I will only prove explicitly that Profile 17 cannot be an equilibrium. We saw in the analysis above that such a profile by candidates could lead parties to adopt one of two Nash equilibria. We cannot know which one will be chosen by parties, but I will show that neither of them can be sustained. Candidates need to form a belief about the NE that will be adopted by parties. On one hand, if candidates believed that parties will choose (l_1, r_1) then precandidate r_2 would expect to lose the nomination; so she would prefer changing her location to zero, which would induce Subgame 3. According to Table 4, this would give r_2 a strictly positive probability of being nominated and winning the election, which is an improvement. So Profile 17 could not be an equilibrium under this belief. On the other hand, if candidates believed that parties will choose (l_2, r_2) then precandidate l_1 would expect to lose the nomination; so she would prefer changing her location to l_2, which would induce Subgame 7. According to Table 4 (and Example 2 in the main text), this would give l_1 a strictly positive probability of being nominated and winning the election, which is an improvement. So Profile 17 could not be an equilibrium under this belief. In sum, this profile cannot be sustained by candidates whichever equilibrium is expected to be chosen by parties subsequently.

All other profiles are studied in a similar way. In particular, the proof that Profile 1 is a NE is the same as in the risk-averse case. From Table 5 we also conclude that Profile 1 is the only solution.

A.4 Proof with risk-neutral parties ($a = 1$)

Once again, we solve the game by backward induction.

- **Third stage**

Sincere voting is a weakly dominant strategy for voters. The proof is the same as with risk-averse parties.

- **Second stage**

In Table 6, we list again all the possible subgames that parties may face, along with their corresponding NE.

Table 6 – Equilibria between parties – risk-neutral case			
		NE without IA3	NE with IA3
Subg. 1	$0 = r_1 = r_2 = -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 2	$0 < r_1 = r_2 = -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 3	$0 = r_1 = r_2 = -l_1 < -l_2$	(l_1, rand) and (l_2, rand)	(l_1, rand)
Subg. 4	$0 < r_1 = r_2 = -l_1 < -l_2$	(l_1, rand)	(l_1, rand)
Subg. 5	$0 = r_1 = r_2 < -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 6	$0 < r_1 = r_2 < -l_1 = -l_2$	$(\text{rand}, \text{rand})$	$(\text{rand}, \text{rand})$
Subg. 7	$0 = r_1 < r_2 = -l_1 = -l_2$	(rand, r_1) and (rand, r_2)	(rand, r_1)
Subg. 8	$0 < r_1 < r_2 = -l_1 = -l_2$	(rand, r_1)	(rand, r_1)
Subg. 9	$0 = r_1 = r_2 < -l_1 < -l_2$	(l_1, rand) and (l_2, rand)	(l_1, rand) and (l_2, rand)
Subg. 10	$0 < r_1 = r_2 < -l_1 < -l_2$	(l_1, rand) and (l_2, rand)	(l_1, rand) and (l_2, rand)

Table 6 – Equilibria between parties – risk-neutral case (continued)			
		NE without IA3	NE with IA3
Subg. 11	$0 = r_1 < r_2 = -l_1 < -l_2$	(l_1, r_1) and (l_1, r_2)	(l_1, r_1)
Subg. 12	$0 < r_1 < r_2 = -l_1 < -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 13	$0 = r_1 < r_2 < -l_1 = -l_2$	(rand, r_2)	(rand, r_2)
Subg. 14	$0 < r_1 < r_2 < -l_1 = -l_2$	(rand, r_2)	(rand, r_2)
Subg. 15	$0 = r_1 < r_2 < -l_1 < -l_2$	(l_1, r_2) and (l_2, r_2)	(l_1, r_2) and (l_2, r_2)
Subg. 16	$0 < r_1 < r_2 < -l_1 < -l_2$	(l_1, r_2) and (l_2, r_2)	(l_1, r_2) and (l_2, r_2)
Subg. 17	$0 = r_1 = -l_1 < r_2 = -l_2$	(l_1, r_1) and (l_1, r_2) and (l_2, r_1) and (l_2, r_2)	(l_1, r_1)
Subg. 18	$0 < r_1 = -l_1 < r_2 = -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 19	$0 = r_1 = -l_1 < r_2 < -l_2$	(l_1, r_1) and (l_1, r_2)	(l_1, r_1)
Subg. 20	$0 < r_1 = -l_1 < r_2 < -l_2$	(l_1, r_1)	(l_1, r_1)

Table 6 – Equilibria between parties – risk-neutral case (continued)			
		NE without IA3	NE with IA3
Subg. 21	$0 = r_1 < -l_1 < r_2 = -l_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)
Subg. 22	$0 < r_1 < -l_1 < r_2 = -l_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)
Subg. 23	$0 = r_1 < -l_1 < r_2 < -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 24	$0 < r_1 < -l_1 < r_2 < -l_2$	(l_1, r_1)	(l_1, r_1)
Subg. 25	$0 = r_1 < -l_1 = -l_2 < r_2$	(rand, r_1)	(rand, r_1)
Subg. 26	$0 < r_1 < -l_1 = -l_2 < r_2$	(rand, r_1)	(rand, r_1)
Subg. 27	$0 = r_1 < -l_1 < -l_2 < r_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)
Subg. 28	$0 < r_1 < -l_1 < -l_2 < r_2$	(l_1, r_1) and (l_2, r_1)	(l_1, r_1) and (l_2, r_1)

As we can see by comparing Table 6 for risk-neutral parties with Table 2 for risk-averse parties, all the subgames are solved the same way except for Subgames 7, 11 and 17.

Let us study subgame 7. Party L does not have a real choice since both of its candidates have adopted indistinguishable platforms. Its unique available strategy is to randomize between l_1 and l_2. On the other hand, party R has a choice between $r_1 = 0$ and $r_2 > 0$. If it nominates r_1 it will win the election over L and the policy implemented will be 0 for sure. If it nominates r_2 it will tie with L and the policy implemented will be a random draw between r_2 and $-r_2$, which represents a lottery with an average policy of zero. This presents a dilemma for R that it would not face if it was risk-averse instead of being risk-neutral. If R was risk-averse, we can see from Table 2 that it would prefer the riskless candidate r_1. However, now that R is risk-neutral, both candidates are equivalent because R is indifferent between obtaining the policy 0 for sure and facing a random draw with an expected policy of 0. Hence, in principle, this subgame has two Nash equilibria in pure strategies which are (rand, r_1) and (rand, r_2). However, only one can survive the assumption IA3, namely (rand, r_1), because r_1 allows L to win the election for sure whereas r_2 would only win with a probability of one half.

The study of subgames 11 and 17 follows a similar same logic.

- **First stage**

In Table 7, we list again all the possible profiles of platforms that candidates may adopt, along with a profitable deviation, if any.

Table 7 – Equilibria between candidates – risk-neutral case			
		Profitable deviation	Is it a Nash equilibrium?
Profile 1	$0 = r_1 = r_2 = -l_1 = -l_2$	None	Yes
Profile 2	$0 < r_1 = r_2 = -l_1 = -l_2$	$r_1 \to r_1 - \varepsilon$	No
Profile 3	$0 = r_1 = r_2 = -l_1 < -l_2$	$l_2 \to 0$	No
Profile 4	$0 < r_1 = r_2 = -l_1 < -l_2$	$l_2 \to l_1 + \varepsilon$	No
Profile 5	$0 = r_1 = r_2 < -l_1 = -l_2$	$r_2 \to r_2 + \varepsilon$	No
Profile 6	$0 < r_1 = r_2 < -l_1 = -l_2$	$r_2 \to r_2 + \varepsilon$	No
Profile 7	$0 = r_1 < r_2 = -l_1 = -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 8	$0 < r_1 < r_2 = -l_1 = -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 9	$0 = r_1 = r_2 < -l_1 < -l_2$	$r_2 \to r_2 + \varepsilon$	No
Profile 10	$0 < r_1 = r_2 < -l_1 < -l_2$	$r_2 \to r_2 + \varepsilon$	No

Table 7 – Equilibria between candidates – risk-neutral case (continued)			
		Profitable deviation	Is it a Nash equilibrium?
Profile 11	$0 = r_1 < r_2 = -l_1 < -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 12	$0 < r_1 < r_2 = -l_1 < -l_2$	$r_2 \to r_2 - \varepsilon$	No
Profile 13	$0 = r_1 < r_2 < -l_1 = -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 14	$0 < r_1 < r_2 < -l_1 = -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 15	$0 = r_1 < r_2 < -l_1 < -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 16	$0 < r_1 < r_2 < -l_1 < -l_2$	$r_1 \to r_2 + \varepsilon$	No
Profile 17	$0 = r_1 = -l_1 < r_2 = -l_2$	$r_2 \to 0$	No
Profile 18	$0 < r_1 = -l_1 < r_2 = -l_2$	$r_2 \to r_1 - \varepsilon$	No
Profile 19	$0 = r_1 = -l_1 < r_2 < -l_2$	$l_2 \to 0$	No
Profile 20	$0 < r_1 = -l_1 < r_2 < -l_2$	$r_2 \to r_1 - \varepsilon$	No

Table 7 – Equilibria between candidates – risk-neutral case (continued)			
		Profitable deviation	Is it a Nash equilibrium?
Profile 21	$0 = r_1 < -l_1 < r_2 = -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 22	$0 < r_1 < -l_1 < r_2 = -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 23	$0 = r_1 < -l_1 < r_2 < -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 24	$0 < r_1 < -l_1 < r_2 < -l_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 25	$0 = r_1 < -l_1 = -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 26	$0 < r_1 < -l_1 = -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 27	$0 = r_1 < -l_1 < -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No
Profile 28	$0 < r_1 < -l_1 < -l_2 < r_2$	$r_2 \to r_1 + \varepsilon$	No

Proving that Profile 1 is an equilibrium follows the same steps as before. And all the other profiles show the same profitable deviations as in Table 3 with risk-averse parties. So I omit any further analysis: the cases analyzed in Tables 2-7 prove the theorem in this chapter.

References

[1] Adams, James, and Samuel Merrill. "Candidate and party strategies in two-stage elections beginning with a primary." American Journal of Political Science 52.2 (2008): 344-359.

[2] Amorós, Pablo, M. Socorro Puy, and Ricardo Martínez. "Closed primaries versus top-two primaries." Public Choice 167, no. 1-2 (2016): 21-35.

[3] Calvert, Randall L. (1985), "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence," *American Journal of Political Science*, Vol. 29, p. 69-95.

[4] Downs, Anthony. 1957. An Economic Theory of Democracy. New York: Harper and Brothers Publishers.

[5] Gerber, Elisabeth R., and Rebecca B. Morton. 1998. “Primary Election Systems and Representation.” Journal of Law, Economics, & Organization 14: 304-324.

[6] Grofman, Bernard, Orestis Troumpounis, and Dimitrios Xefteris 2016. "Electoral competition with primaries and quality asymmetries." Lancaster University Management School. Economics Working Paper Series 2016/016.

[7] Hall, Andrew B. and Snyder, James M., Information and Wasted Votes: A Study of U.S. Primary Elections (August 10, 2015). Quarterly Journal of Political Science, 10(4): 433-459.

[8] Hortala-Vallve, Rafael, and Hannes Mueller. "Primaries: the unifying force." Public Choice 163, no. 3-4 (2015): 289-305.

[9] Hummel, Patrick. "Candidate strategies in primaries and general elections with candidates of heterogeneous quality." Games and Economic Behavior 78 (2013): 85-102.

[10] Jackson, Matthew O., Laurent Mathevet, and Kyle Mattes. "Nomination processes and policy outcomes." Quarterly Journal of Political Science 2, no. 1 (2007): 67-92.

[11] Kselman, Daniel M. 2015. "A median activist theorem for two-stage spatial models." In The Political Economy of Governance: Institutions, Political Performance and Elections, edited by Norman Schofield and Gonzalo Caballero. Switzerland: Springer (May), 193-210.

[12] Masket, Seth. 2016. The Inevitable Party: Why Attempts to Kill the Party System Fail and How they Weaken Democracy. New York: Oxford University Press.

[13] Sandri, Giulia, and Antonella Seddone. 2015. “Introduction: Primary Elections Across the World.” In Party Primaries in Comparative Perspective, edited by Giulia Sandri, Antonella Seddone and Venturino Fulvio, 1-20. Farnham: Ashgate.

[14] Serra, Gilles. 2011. “Why Primaries? The Party’s Tradeoff between Policy and Valence.” Journal of Theoretical Politics 23 (1), January: 21-51.

[15] Serra, Gilles. 2015. “No Polarization in Spite of Primaries: A Median Voter Theorem with Competitive Nominations.” In The Political Economy of Governance: Institutions, Political Performance and Elections, edited by Norman Schofield and Gonzalo Caballero. Switzerland: Springer (May), 211-229.

[16] Serra, Gilles. 2017. “Contagious sincerity: When should we expect partisan primaries in one party to induce partisan primaries in the rival party?” CIDE Working Papers DTEP-291 (June).

[17] Serra, Gilles. 2018. “Primaries, Conventions, and Other Methods for Nominating Candidates: How Do They Matter?” In The Oxford Handbook of Public Choice, edited by Roger Congleton, Bernard Grofman, and Stefan Voigt. Oxford: Oxford University Press (forthcoming).

[18] Snyder Jr., J. M., & Ting, M. M. (2011). Electoral selection with parties and primaries. American Journal of Political Science, 55(4), 782-796.

[19] Wittman, Donald (1973), "Parties as Utility Maximizers", *American Political Science Review,* Vol. 67, p. 490-498.

[20] Woon, J. (2016). “Primaries, strategic voting, and candidate polarization.” Manuscript. Available at http://recursos.march.es/web/ceacs/actividades/pdf/Woon.pdf.